

Name: _____

Lecture Section _____

Ma 221

Exam IIIA

13S

I pledge my honor that I have abided by the Stevens Honor System. _____

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Score on Problem #1a _____

#1b _____

#2 _____

#3 _____

#4 _____

Total Score _____

Note: A table of Laplace Transforms is given at the end of the exam.

Name: _____

Lecture Section ____

1a (10 pts.) Let $F(s) = \hat{f}(s) = \mathcal{L}\{f(t)\}$. Show that

$$\mathcal{L}\{e^{-at}f(t)\} = F(s + a) = \hat{f}(s + a)$$

1b (15 pts.) Find

$$\mathcal{L}^{-1}\left(\frac{1}{s^3 + s}\right)$$

Name: _____

Lecture Section ____

2 (25 pts.) Use Laplace Transforms to solve the initial value problem

$$y'' + y' - 2y = 4 \quad y(0) = 2 \quad y'(0) = 1$$

Name: _____

Lecture Section ____

3 (25 pts.) Find the first 5 nonzero terms of the power series solution about $x = 0$ for the DE:

$$y'' - 2xy' + 2y = 0$$

Be sure to give the recurrence relation.

Name: _____

Lecture Section ____

4 (25 pts.) Find all eigenvalues (λ) and the corresponding eigenfunctions for the boundary value problem

$$y'' - 2y + \lambda y = 0 \quad y'(0) = y'(\pi) = 0$$

Be sure to consider all values of λ .

Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}\{f\}(s) = \hat{f}(s)$		
$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	$n \geq 1$	$s > 0$
e^{at}	$\frac{1}{s-a}$		$s > a$
$\sin bt$	$\frac{b}{s^2 + b^2}$		$s > 0$
$\cos bt$	$\frac{s}{s^2 + b^2}$		$s > 0$
$e^{at}f(t)$	$\mathcal{L}\{f\}(s-a)$		
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$		