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Ma 221 Exam IIIB 14S

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Score on Problem #1a ________

#1b________

#2a ________

#2b ________

#3 ________

#4 ________

Total Score ________

I pledge my honor that I have abided by the Stevens Honor System.

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Note: A table of Laplace Transforms is given at the end of the exam.

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1a (10 **pts**.) Use the definition of the Laplace transform to determine the Laplace transform of $f(t) = e^{at}$, where a is a constant.$\mathcal{L}\{e^{at}\}$.

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1b (15 **pts**.) Determine

$$\mathcal{L}^{-1}\left\{\frac{3s+5}{s^2+4s+13}\right\}.$$

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2a (15 **pts**.) Consider the initial value problem

$$y'' + y = \cos 2t \quad y(0) = 2 \;\; y'(0) = 1.$$

Let $Y(s) = \mathcal{L}\{y\}(s)$. Use Laplace transforms to show that

$$Y(s) = \frac{s}{(s^2+4)(s^2+1)} + \frac{2s}{s^2+1} + \frac{1}{s^2+1}.$$

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2b (15 **pts**.) Find the solution to the initial problem above, namely,

$$y'' + y = \cos 2t \quad y(0) = 2 \;\; y'(0) = 1$$

by finding

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)(s^2+1)} + \frac{2s}{s^2+1} + \frac{1}{s^2+1}\right\}$$

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3 (25 **pts**.) Find the first 5 nonzero terms of the power series solution about $x = 0$ for the DE:

$$y'' + 3xy' - 3y = 0$$

Be sure to give the recurrence relation.

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4 (25 **pts**.) Consider the boundary value problem

$$x^2 y'' + 3xy' + \lambda y = 0 \quad y(1) = y(e^2) = 0.$$

Assuming the solution is of the form $y = x^m$ leads to

$$m = -1 \pm \sqrt{1 - \lambda}\,.$$

Consider only the case $1 - \lambda < 0$. Find the eigenvalues (λ) and the corresponding eigenfunctions for the boundary value problem for this case.

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Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}\{f\}(s) = \hat{f}(s)$		
$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	$n \geq 1$	$s > 0$
e^{at}	$\frac{1}{s-a}$		$s > a$
$\sin bt$	$\frac{b}{s^2 + b^2}$		$s > 0$
$\cos bt$	$\frac{s}{s^2 + b^2}$		$s > 0$
$e^{at}f(t)$	$\mathcal{L}\{f\}(s-a)$		
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$		