Ma 221 - Exam I review

First Order Differential Equations

Separable Equations

\[
\frac{dy}{dx} = g(x)p(y)
\]

\[
h(y)dy = g(x)dx
\]

\[
\int h(y)dy = \int g(x)dx
\]

Linear Equations

\[
\frac{dy}{dx} + p(x)y = q(x)
\]

Integrating Factor

\[
IF = e^{\int p(x)dx}
\]

\[
e^{\int p(x)dx} \left( \frac{dy}{dx} + p(x)y \right) = e^{\int p(x)dx} [q(x)]
\]

\[
\frac{d}{dx} \left( e^{\int p(x)dx} y \right) = e^{\int p(x)dx} [q(x)]
\]

\[
e^{\int p(x)dx} y = \left( e^{\int p(x)dx} [q(x)] \right) dx
\]

\[
y = e^{-\int p(x)dx} \int \left( e^{\int p(x)dx} [q(x)] \right) dx
\]

Exact Equations

\[
M(x,y)dx + N(x,y)dy = 0
\]

Test for exactness

\[
\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}
\]

When test is passed, there is \( F(x,y) \) such that

\[
M = \frac{\partial F}{\partial x} \quad \text{and} \quad N = \frac{\partial F}{\partial y}
\]

Find \( F(x,y) \). Solution is

\[
F(x,y) = c
\]
Bernoulli D.E.

\[ \frac{dy}{dx} + p(x)y = q(x)y^n \]

\[ y^{-n} \frac{dy}{dx} + p(x)y^{1-n} = q(x) \]

Substitution

\[ z = y^{1-n} \]

\[ \frac{dz}{dx} = (1 - n)y^{-n} \frac{dy}{dx} \]

yields a linear d.e. in \( z \).

In all cases, the arbitrary constant resulting from integration is used to satisfy any initial condition.