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Ma 221 Exam IIIB 15S

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Score on Problem #1a ________

#1b________

#2a ________

#2b ________

#3 ________

#4 ________

Total Score ________

I pledge my honor that I have abided by the Stevens Honor System.

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Note: A table of Laplace Transforms is given at the end of the exam.

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1a (10 **pts**.)

$$f(t) = \begin{cases} 0 & if \quad 0 \le t < 3 \\ e^{2t} & if \quad 3 \le t \end{cases}$$

Use the definition of the Laplace transform to determine the Laplace transform of $f(t)$. For which values of s are your results valid?

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1b (15 **pts**.) Determine

$$\mathcal{L}^{-1}\left\{\frac{3s+3}{s^2+6s+13}\right\}$$

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2a (15 **pts**.) Consider the initial value problem

$$y'' + 3y' + 2y = 12e^{-2t} \quad y(0) = 2 \;\; y'(0) = -8$$

Let $Y(s) = \mathcal{L}\{y\}(s)$. Use Laplace transforms to show that

$$Y(s) = \frac{12}{(s+1)(s+2)^2} + \frac{2s-2}{(s+1)(s+2)}.$$

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2b (10 **pts**.) We find the solution to the initial problem above, namely,

$$y'' + 3y' + 2y = 12e^{-2t} \quad y(0) = 2 \;\; y'(0) = -8$$

by finding

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{12}{(s+1)(s+2)^2} + \frac{2s-2}{(s+1)(s+2)}\right\}.$$

This requires use of partial fractions to find an equivalent expression for $Y(s)$ made up of terms in the Laplace transform table. Give the form of this partial fractions decomposition for $Y(s)$. (You do not have to find the coefficients.)

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3 (25 **pts**.) Find the first 5 nonzero terms of the power series solution about $x = 0$ for the DE:

$$y'' - 2xy = 0$$

Be sure to give the recurrence relation.

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4 (25 **pts**.) Find all eigenvalues (λ) and the corresponding eigenfunctions for the boundary value problem

$$y'' + (-2 + \lambda)y = 0 \qquad y'(0) = y'(\pi) = 0$$

Be sure to consider all values of λ.

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Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}\{f\}(s) = \hat{f}(s)$		
1	$\frac{1}{s}$		$s > 0$
t^n	$\dfrac{n!}{s^{n+1}}$	$n \geq 1$	$s > 0$
e^{at}	$\dfrac{1}{s-a}$		$s > a$
$\sin bt$	$\dfrac{b}{s^2 + b^2}$		$s > 0$
$\cos bt$	$\dfrac{s}{s^2 + b^2}$		$s > 0$
$e^{at}t^n$	$\dfrac{n!}{(s-a)^{n+1}}$	$n \geq 1$	$s > a$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2 + b^2}$		$s > a$
$e^{at}\cos bt$	$\dfrac{s}{(s-1)^2 + b^2}$		$s > a$
$e^{at}f(t)$	$\mathcal{L}\{f\}(s-a)$		
$t^n f(t)$	$(-1)^n \dfrac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$		

Properties of Laplace Transforms

$\mathcal{L}\{f+g\} = \mathcal{L}\{f\} + \mathcal{L}\{f\}$
$\mathcal{L}\{cf\} = c\mathcal{L}\{f\}$
$\mathcal{L}\{f'\}(s) = s\mathcal{L}\{f\}(s) - f(0)$
$\mathcal{L}\{f''\}(s) = s^2\mathcal{L}\{f\}(s) - sf(0) - f'(0)$