Ma 221 **Final Exam** **18 May 2015**

Print Name: ________________________________ **Lecture Section**: _________

Lecturer__________________

This exam consists of 7 problems. You are to solve all of these problems. The point value of each problem is indicated. The total number of points is 200.

If you need more work space, continue the problem you are doing on the **other side of the page it is on**. Be sure that you do all problems. **There are tables giving Laplace transforms and integrals at the end of the exam**.

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

I pledge my honor that I have abided by the Stevens Honor System.

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Score on Problem #1a ________/10 #1b ________/10 #1c ________/10

#2a ________/ 8 #2b________/ 8 #2c ________/14

#3a ________/15 #3b ________/15

#4a ________/ 9 #4b ________/ 6 #4c ________/15

#5a ________/15 #5b ________/10

#6a________/ 4 #6b ________/ 9 #6c ________/ 4 #6d ________/ 8

#7a ________/15 #7b ________/15

Total Score _________/200

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1.Solve the following initial value problems.

(a) (10 pts)

$$x\frac{dv}{dx} = \frac{1+4v^2}{4v}, \quad v(1) = 0.$$

(b) (10 pts)

$$\frac{dy}{dx} + 4xy = 8x, \qquad y(0) = 0$$

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1 (c) (10 pts)

$$(2xy - 3x^2)dx + \left(x^2 - \frac{2}{y^3}\right)dy = 0, \qquad y(1) = -1$$

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2. (a) (8 pts) Find a general solution of

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 0, \qquad \text{for } -\infty < x < \infty.$$

2(b) (8 pts.) Find a general solution of

$$t^2\frac{d^2y}{dt^2} + 3t\frac{dy}{dt} + 5y = 0, \qquad \text{for } t > 0.$$

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2(c) (14 pts.) Use the method of undetermined coefficients to find a general solution of

$$L[y] = y'' + 5y' + 4y = e^{-t} + 4te^{-2t}.$$

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3 (a) (15 pts.) Consider the differential equation

$$L[y] = t^2 \frac{d^2y}{dt^2} + 4t\frac{dy}{dt} + 2y = 4\ln t, \qquad \textit{for} \;\; t > 1.$$

Solutions to the homogeneous equation are $y_1(t) = \frac{1}{t}$ and $y_2(t) = \frac{1}{t^2}$.

(i) Use the Wronskian, $W[y_1, y_2](t)$, to verify that y_1 and y_2 are linearly independent solutions on the interval $t > 1$.

(ii) Find a particular solution, $y_p(t)$ satisfying

$$L[y] = t^2 \frac{d^2y}{dt^2} + 4t\frac{dy}{dt} + 2y = 4\ln t.$$

(iii) Find a general solution to the equation.

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3 (b) (15 pts.) Classify each of the following differential equations as linear or nonlinear. If nonlinear, identify all terms that make the equation nonlinear. (In all cases, consider y to be the dependent variable and t the independent variable.)

	Equation	Linear/nonlinear	Nonlinear terms
(i)	$y'' + 3y' + 4\sin(y) = 0$		
(ii)	$y'' + 3y' + 4y = \cos(4t)$		
(iii)	$t^2y'' - ty' + y = \ln(t)/t$		
(iv)	$e^{-t}dy + (t^2y - \sin t)dt = 0$		
(v)	$y'' + 2yy' + 4y = 0$		
(vi)	$ty' + t^3y^2 = 4t^2$		

4. (a) (9 pts.) Find the inverse Laplace transform for

$$F(s) = \frac{2s+1}{(s+2)^3}$$

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4 (b) (6pts.) Determine the Laplace transform for $f(t) = t\sin(3t)$.

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4 (c) (15 pts.) Solve using Laplace Transforms:

$$L[y] = y'' + 2y' + 5y = te^{-2t}, \qquad y(0) = 0, \ \ y'(0) = 1.$$

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5. (a) (15 pts.) Find the first five non-zero terms of the Fourier *cosine* series for the function

$$f(x) = \begin{cases} 0, & 0 < x < \frac{1}{2} \\ x, & \frac{1}{2} < x < 1 \end{cases} .$$

5(b) (10 pts.) To what value does the Fourier series of 5a converge at each of the following points?

(i) $x = -\frac{3}{4}$ (ii) $x = 0$ (iii) $x = \frac{1}{2}$

(iv) $x = 1$ (v) $x = \frac{3}{2}$

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6 (25 pts.) Consider the following initial-boundary value problem for the heat equation.

$$
\begin{array}{lrcll}
\text{PDE} & u_t &=& 3u_{xx}, & 0 < x < \pi, \quad t > 0 \\
\text{BC1} & u(0,t) &=& 0, & t > 0 \\
\text{BC2} & u_x(\pi,t) &=& 0, & t > 0 \\
\text{IC} & u(x,0) &=& 3\sin\frac{x}{2} - \sin\frac{11x}{2} + 7\sin\frac{19x}{2}, & 0 < x < \pi
\end{array}
\qquad (1)
$$

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6 (a) Let the solution be $u(x,t) = X(x)T(t)$.. Use the method of separation of variables and the boundary conditions to obtain an eigenvalue problem for $X(x)$ and a differential equation for $T(t)$.

6 (b) Solve the eigenvalue problem.

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6 (c) For each eigenvalue, solve the corresponding differential equation for $T(t)$. For each eigenvalue give the corresponding solution to the heat equation.

6 (d) Give the formal series solution to the initial-boundary value problem. Apply the initial condition to determine the coefficients of the series. Give the final solution to the problem.

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7 (a) (15 pts.) Find the power series solution to the initial value problem

$$y'' + 2xy' - 3y = 0, \qquad y(0) = 1, \ \ y'(0) = 0.$$

Be sure to give the recurrence relation for the coefficients of the power series. Give the first five nonzero terms of the solution.

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7 (b) (15 pts.) Find the eigenvalues and eigenfunctions for

$$y'' + \lambda y = 0, \qquad 0 < x < 5$$
$$y'(0) = 0$$
$$y(5) = 0$$

Be sure to consider the cases $\lambda < 0, \lambda = 0,$ and $\lambda > 0.$

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Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}\{f\}(s) = \hat{f}(s)$		
1	$\frac{1}{s}$		$s > 0$
t^n	$\dfrac{n!}{s^{n+1}}$	$n \geq 1$	$s > 0$
e^{at}	$\dfrac{1}{s-a}$		$s > a$
$\sin bt$	$\dfrac{b}{s^2 + b^2}$		$s > 0$
$\cos bt$	$\dfrac{s}{s^2 + b^2}$		$s > 0$
$e^{at}t^n$	$\dfrac{n!}{(s-a)^{n+1}}$	$n \geq 1$	$s > a$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2 + b^2}$		$s > a$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2 + b^2}$		$s > a$
$e^{at}f(t)$	$\mathcal{L}\{f\}(s-a)$		
$t^n f(t)$	$(-1)^n \dfrac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$		

Properties of Laplace Transforms

$\mathcal{L}\{f+g\} = \mathcal{L}\{f\} + \mathcal{L}\{f\}$
$\mathcal{L}\{cf\} = c\mathcal{L}\{f\}$
$\mathcal{L}\left\{f'\right\}(s) = s\mathcal{L}\{f\}(s) - f(0)$
$\mathcal{L}\left\{f''\right\}(s) = s^2\mathcal{L}\{f\}(s) - sf(0) - f'(0)$

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Table of Integrals

$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int x \cos bx dx = \frac{1}{b^2}(\cos bx + bx \sin bx) + C$
$\int x \sin bx dx = \frac{1}{b^2}(\sin bx - bx \cos bx) + C$
$\int \sec x dx = \ln\lvert\sec x + \tan x\rvert + C$
$\int \ln t dt = t \ln t - t + C$
$\int t \ln t dt = \frac{1}{2}t^2 \ln t - \frac{1}{4}t^2 + C$
$\int t^2 \ln t dt = \frac{1}{3}t^3 \ln t - \frac{1}{9}t^3 + C$
$\int t \ln^2 t dt = \frac{1}{4}t^2(2\ln^2 t - 2\ln t + 1) + C$
$\int \frac{\ln t}{t} dt = \frac{1}{2}\ln^2 t + C$