Ma 221 **Final Exam**
12/16/14

Print Name: ____ **Lecture Section**: _________

This exam consists of 8 problems. You are to solve all of these problems. The point value of each problem is indicated. The total number of points is 200.

If you need more work space, continue the problem you are doing on the **other side of the page it is on**. Be sure that you do all problems.

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

I pledge my honor that I have abided by the Stevens Honor System.

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There are tables giving Laplace transforms and integrals at the end of the exam.

Score on Problem #1a ________

#1b ________

#1c ________

#2a ________

#2b ________

#3a ________

#3b ________

#4a ________

#4b ________

#5a ________

#5b ________

#6 ________

#7a ________

#7b ________

#8a ________

#8b ________

Total Score _________

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1.

(a) (8 pts) Solve

$$\frac{dy}{dx} = \frac{\sin x}{2e^y} \quad y(0) = 0.$$

(b) (7 pts) Solve

$$(2x\cos y + 1)dx + \left(-x^2 \sin y + 2y\right)dy = 0.$$

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1 (c) (10 pts) Find a general solution of

$$x^2 y'' + 3xy' + 5y = 0.$$

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2. (a) (12 pts) Find a general solution of

$$y'' - 2y' = 4x + 2 - 10\sin x.$$

2(b) (13 pts.) Find a general solution of

$$y'' - 2y' + y = e^x \ln x.$$

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3. (a) (10 pts.) Let

$$g(t) = \begin{cases} t & \text{for } 0 < t < 1 \\ e^t & \text{for } 1 < t < \infty \end{cases}.$$

Use the definition of the Laplace transform to find $\mathcal{L}\{g(t)\}$.

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(b) (15 pts.) Solve using Laplace Transforms:

$$y'' - 2y' - 3y = 16e^{-t} \qquad y(0) = 1, \;\; y'(0) = 3.$$

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4.) a.) (10 pts.) Use separation of variables, $u(x,t) = X(x)T(t)$, to find two ordinary differential equations which $X(x)$ and $T(t)$ must satisfy to be a solution of

$$e^{x-t}\frac{\partial^2 u}{\partial x^2} - (x-3)^2 t^5 \frac{\partial u}{\partial t} = 0.$$

Note: Do **not** solve these ordinary differential equations.

b.) (15 pts.) Find

$$\mathcal{L}^{-1}\left\{ \frac{2s^3 + 5s^2 + 6s + 7}{\left(s^2 - 1\right)\left(s^2 + 4s + 5\right)} \right\}.$$

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5. (a) (15 pts.) Find the first five non-zero terms of the Fourier *sine* series for the function

$$f(x) = \begin{cases} 0 & 0 < x < \pi \\ 1 & \pi < x < 2\pi \end{cases}$$

5(b) (10 pts.) To what value does the Fourier series of 5a converge at each of the following points?

(i) $x = -\frac{3\pi}{2}$

(ii) $x = 0$

(iii) $x = \pi$

(iv) $x = \frac{3\pi}{2}$

(v) $x = \frac{5\pi}{2}$

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6 (25 pts.) Solve the following initial-boundary value problem.

$$\begin{array}{lll} \text{PDE} & u_t = 3u_{xx}, & 0 < x < 4, \quad t > 0 \\ \text{BCs} & u_x(0,t) = 0 & u_x(4,t) = 0 \\ \text{IC} & u(x,0) = \cos\left(\frac{\pi}{2}x\right) - 7\cos\left(\frac{3\pi}{4}x\right) + 5\cos\left(\frac{3\pi}{2}x\right) & \end{array}$$

You must derive the solution. Your solution should not have any arbitrary constants in it. Show **all** steps.

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7. (a) (13 pts.) Find a general solution of

$$y'' + 2y' + y = \frac{e^{-x}}{x^2}$$

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7 (b) (12 pts.) Find the power series solution to

$$y'' + xy' - 2y = 0$$

near $x = 0$. Be sure to give the recurrence relation for the coefficients of the power series. Indicate the two linearly independent solutions and give the first six nonzero terms of the solution.

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8 (a) (15 pts.) Find the eigenvalues and eigenfunctions for

$$y'' + \lambda y = 0 \qquad 0 < x < 1$$
$$y'(0) = y(1) = 0$$

Be sure to consider the cases $\lambda < 0, \lambda = 0,$ and $\lambda > 0$.

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8(b) (10 pts.) Solve the initial value problem

$$\frac{dy}{dx} + y\tan x = \frac{\sec x}{y^2} \quad y(0) = 1$$

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Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$		
$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	$n \geq 1$	$s > 0$
e^{at}	$\frac{1}{s-a}$		$s > a$
$\sin bt$	$\frac{b}{s^2+b^2}$		$s > 0$
$\cos bt$	$\frac{s}{s^2+b^2}$		$s > 0$
$e^{at}f(t)$	$\mathcal{L}\{f\}(s-a)$		
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$		

Table of Integrals

$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int x \cos bx dx = \frac{1}{b^2}(\cos bx + bx \sin bx) + C$
$\int x \sin bx dx = \frac{1}{b^2}(\sin bx - bx \cos bx) + C$
$\int \tan u du = -\ln(\cos u) + C$
$\int \tan^2 u du = \tan u - u + C$
$\int \sec u du = \ln(\sec u + \tan u) + C$
$\int \sec^2 u du = \tan u + C$
$\int \sec^3 u du = \frac{1}{2}[\sec u \tan u + \ln(\sec u + \tan u] + C$
$\int \sec^4 u du = \tan u + \frac{1}{3}\tan^3 u + C$
$\int \ln u du = u \ln u - u + C$
$\int u \ln u du = \frac{1}{2}u^2 \ln u - \frac{1}{4}u^2 + C$
$\int u^2 \ln u du = \frac{1}{3}u^3 \ln u - \frac{1}{9}u^3 + C$
$\int \frac{\ln u}{u} du = \frac{1}{2}\ln^2 u + C$