

MA 221 Homework Solutions

Due date: March 25, 2014

Section 8.2 pg. 434 # 1, 2, 5, 6

(Underlined problems are to be handed in)

1.)

$$\sum_{n=0}^{\infty} \frac{2^{-n}}{n+1} (x-1)^n$$

$$\lim_{n \rightarrow \infty} \left(\frac{a_{n+1}}{a_n} \right) = \lim_{n \rightarrow \infty} \frac{2^{-(n+1)} / (n+2)}{2^{-n} / (n+1)} = \frac{1}{2} = L$$

$$p = \frac{1}{L} = 2$$

The endpoints of the interval of convergence are

$$x_1 = x_0 + p = 1 + 2 = 3$$

$$x_2 = x_0 - p = 1 - 2 = -1$$

At x_1

$$\sum_{n=0}^{\infty} \frac{2^{-n}}{n+1} (3-1)^n = \sum_{n=0}^{\infty} \frac{1}{n+1} = \infty$$

At x_2

$$\sum_{n=0}^{\infty} \frac{2^{-n}}{n+1} (-1-1)^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} < \infty$$

Therefore, the set of convergence is

$$[-1, 3)$$

2.)

$$\sum_{n=0}^{\infty} \frac{3^n}{n!} x^n$$

$$\lim_{n \rightarrow \infty} \left(\frac{3^{n+1} x^{n+1}}{(n+1)!} * \frac{n!}{3^n x^n} \right) = \lim_{n \rightarrow \infty} \frac{3x}{n+1} < 1$$

The set of convergence is

$$(-\infty, \infty)$$

5.)

$$\sum_{n=1}^{\infty} \frac{3}{n^3} (x-2)^n$$

$$\lim_{n \rightarrow \infty} \left(\frac{3/(n+1)^3}{3/n^3} \right) = \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^3 = 1$$

$$p = \frac{1}{L} = 1$$

At $x = 3$ and $x = 1$

$$\sum_{n=0}^{\infty} \frac{3}{n^3} \text{ and } \sum_{n=0}^{\infty} \frac{3(-1)^n}{n^3}$$

The set of convergence is the closed interval

$$[1, 3]$$

6.) $\sum_{n=0}^{\infty} \frac{(n+2)!}{n!} (x+2)^n$

$$\lim_{n \rightarrow \infty} \left(\frac{(n+3)!}{(n+1)!} * \frac{n!}{(n+2)!} \right) = 1$$

$$\sum_{n=0}^{\infty} \frac{(n+2)!}{n!} (-1)^n \rightarrow \text{diverges}$$

$$\sum_{n=0}^{\infty} \frac{(n+2)!}{n!} (1)^n \rightarrow \text{diverges}$$

Therefore, the set of convergence is

$$(-1, -3)$$