

# MA 221 Homework Solutions

## Due date: March 4, 2014

**7.2 pg. 360 # 1, 7, 9, 10, 15, 16, 17, 19**

(Underlined problems are to be handed in)

In problems 1, 7 and 9, use Definition 1 to determine the Laplace transform of the given function.

1.)  $t$

$$\begin{aligned} L\{t\}(s) &= \int_0^\infty e^{-st} dt \\ \lim_{N \rightarrow \infty} \int t d\left(-\frac{e^{-st}}{s}\right) &= \lim_{N \rightarrow \infty} \left[ \left(-\frac{te^{-st}}{s}\right)_0^N + \frac{1}{s} \int_0^N e^{-st} dt \right] \\ \lim_{N \rightarrow \infty} \left[ \left(-\frac{te^{-st}}{s}\right)_0^N + \left(\frac{e^{-st}}{s^2}\right)_0^N \right] &= \lim_{N \rightarrow \infty} \left[ -\frac{Ne^{-sN}}{s} + 0 - \frac{e^{-sN}}{s^2} + \frac{1}{s^2} \right] \\ &= \frac{1}{s^2}, s > 0, e^{-sN} \rightarrow 0 \text{ and } Ne^{-sN} = \frac{N}{e^{sN}} \rightarrow 0 \text{ as } N \rightarrow \infty \end{aligned}$$

7.)  $e^{2t} \cos 3t$

$$\begin{aligned} L\{e^{2t} \cos 3t\}(s) &= \int_0^\infty e^{-st} e^{2t} \cos 3t dt = \int_0^\infty e^{(2-s)t} \cos 3t dt \\ \lim_{N \rightarrow \infty} \left[ \frac{e^{(2-s)t}((2-s)\cos 3t + 3\sin 3t)}{(2-s)^2 + 9} \right]_0^N &= \lim_{N \rightarrow \infty} \frac{e^{(2-s)N}[(2-s)\cos 3N + 3\sin 3N] - (2-s)}{(2-s)^2 + 9} \\ &\quad \frac{s-2}{(s-2)^2 + 9}, s > 2 \end{aligned}$$

9.)

$$f(t) = \begin{cases} 0, & 0 < t < 2 \\ t, & 2 \leq t \end{cases}$$

$$\begin{aligned} L\{f(t)\}(s) &= \int_0^\infty e^{-st} f(t) dt = \int_0^2 e^{-st} \cdot 0 dt + \int_2^\infty t e^{-st} dt = \int_2^\infty t e^{-st} dt \\ \int_2^\infty t e^{-st} dt &= \lim_{N \rightarrow \infty} \left[ \left(-\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2}\right) \right]_2^N = \lim_{N \rightarrow \infty} \left[ -\frac{Ne^{-sN}}{s} - \frac{e^{-sN}}{s^2} + \frac{2e^{-2s}}{s} + \frac{e^{-2s}}{s^2} \right]_0^N \\ \frac{2e^{-2s}}{s} + \frac{e^{-2s}}{s^2} &= e^{-2s} \left( \frac{2}{s} + \frac{1}{s^2} \right) \end{aligned}$$

$$= e^{-2s} \left( \frac{2s+1}{s^2} \right), s > 0$$

10.)

$$\begin{aligned} f(t) &= \begin{cases} 1-t, & 0 < t < 1 \\ 0, & 1 \leq t \end{cases} \\ L\{f(t)\}(s) &= \int_0^\infty e^{-st} f(t) dt = \int_0^1 e^{-st} \cdot (1-t) dt + \int_1^\infty e^{-st} \cdot 0 dt = \int_0^1 (e^{-st} - te^{-st}) dt \\ &\quad \int_0^1 (e^{-st} - te^{-st}) dt = \left[ \frac{1}{s^2} e^{-st} (st - s + 1) \right]_0^1 \\ &\quad \frac{1}{s^2} e^{-s} (s - s + 1) - \left[ \frac{1}{s^2} (-s + 1) \right] = \frac{e^{-s}}{s^2} + \frac{1}{s} - \frac{1}{s^2} \\ &= \frac{1}{s} + \frac{e^{-s} - 1}{s^2}, \text{ for all } s \end{aligned}$$

In problems 15, 16 and 17, use the Laplace transform table and the linearity of the Laplace transform to determine the following transforms.

15.)  $L\{t^3 - te^t + e^{4t} \cos t\}$

$$\begin{aligned} L\{t^3 - te^t + e^{4t} \cos t\}(s) &= L\{t^3\}(s) - L\{te^t\}(s) + L\{e^{4t} \cos t\}(s) \\ &= \frac{3!}{s^{3+1}} - \frac{1!}{(s-1)^{1+1}} + \frac{s-4}{(s-4)^2 + 1^2} \\ &= \frac{6}{s^4} - \frac{1}{(s-1)^2} + \frac{s-4}{(s-4)^2 + 1}, s > 4 \end{aligned}$$

16.)  $L\{t^2 - 3t - 2e^{-t} \sin 3t\}$

$$\begin{aligned} L\{t^2 - 3t - 2e^{-t} \sin 3t\}(s) &= L\{t^2\}(s) - 3L\{t\}(s) + 2L\{e^{-t} \sin 3t\}(s) \\ &= \frac{2!}{s^{2+1}} - 3\left(\frac{1!}{s^{1+1}}\right) - 2\left(\frac{3}{(s-(-1))^2 + 3^2}\right) \\ &= \frac{2}{s^3} - \frac{3}{s^2} - \frac{6}{(s+1)^2 + 9}, s > 0 \end{aligned}$$

17.)  $L\{e^{3t} \sin 6t - t^3 + e^t\}$

$$\begin{aligned} L\{e^{3t} \sin 6t - t^3 + e^t\}(s) &= L\{e^{3t} \sin 6t\}(s) - L\{t^3\}(s) + L\{e^t\}(s) \\ &= \frac{6}{(s-3)^2 + 6^2} - \frac{3!}{s^{3+1}} + \frac{1}{s-1} \\ &= \frac{6}{(s-3)^2 + 36} - \frac{6}{s^4} + \frac{1}{s-1}, s > 3 \end{aligned}$$

19)  $L\{t^4 e^{5t} - e^t \cos \sqrt{7}t\}$

$$\begin{aligned}
L\{t^4 e^{5t} - e^t \cos \sqrt{7} t\}(s) &= L\{t^4 e^{5t}\}(s) - L\{e^t \cos \sqrt{7} t\}(s) \\
&= \frac{4!}{(s-5)^{4+1}} - \frac{s-1}{(s-1)^2 + (\sqrt{7})^2} \\
&= \frac{24}{(s-5)^5} - \frac{s-1}{(s-1)^2 + 7}
\end{aligned}$$