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Ma 221 **Final Exam Solutions** **12/20/04**

Print Name: _________________________________ **ID**: __________________________

Lecture Section: _________

I pledge my honor that I have abided by the Stevens Honor System.

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This exam consists of 7 problems. The point value of each problem is indicated. The total number of points is 175, which will be scaled to 200 points after grading.

If you need more work space, continue the problem you are doing on the **other side of the page it is on**. Be sure that you do all problems.

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Score on Problem #1 ________

#2 ________

#3 ________

#4 ________

#5 ________

#6 ________

#7 ________

Total Score ________

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1. Solve the initial value problem

(a) (8 pts)

$$y' + 2ty = 4t, \;\; y(0) = 5$$

Solution: This is a first order linear DE with $P(t) = 2t$. The integrating factor is $e^{\int 2tdt} = e^{t^2}$. Multiplying the DE by e^{t^2} leads to

$$\frac{d}{dt}\left(e^{t^2}y\right) = 4te^{t^2}$$

Integrating we have

$$e^{t^2}y = 2e^{t^2} + C$$

or

$$y(t) = 2 + Ce^{-t^2}$$

The initial condition implies

$$y(0) = 2 + C = 5$$

so $C = 3$ and

$$y(t) = 2 + 3e^{-t^2}$$

Alternative solution: This equation is also separable, since it can be written as

$$\frac{dy}{dt} = 4t - 2ty = 2t(2 - y)$$

or

$$\frac{dy}{(2-y)} = 2tdt$$

so

$$-\ln(2-y) = t^2 + k$$

or

$$2 - y = e^{-t^2-k} = c_1e^{-t^2}$$

Thus

$$y = 2 + Ce^{-t^2}$$

as before so that $C = 3$ and

$$y(t) = 2 + 3e^{-t^2}$$

(b) (7 pts) Solve

$$1 + y^2 + 2(t+1)y\frac{dy}{dt} = 0$$

Solution: We rewrite the equation at

$$\left(1 + y^2\right)dt + 2(t+1)ydy = 0$$

The $M = 1 + y^2$ and $N = 2(t+1)y$ so

$$M_y = N_t = 2y$$

and the equation is exact.

There exists a function $f(x, y)$ such that

$$f_t = M = 1 + y^2 \;\text{ and }\; f_y = N = 2(t+1)y$$

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Integrating f_t with respect to t gives

$$f = \left(1+y^2\right)t + g(y)$$

so

$$f_y = 2yt + g'(y) = 2ty + 2y$$

Thus

$$g(y) = y^2 + C$$

and

$$f = \left(1+y^2\right)t + y^2 + C$$

so the solution is given by

$$\left(1+y^2\right)t + y^2 = k$$

Alternative solution: The equation

$$1 + y^2 + 2(t+1)y\frac{dy}{dt} = 0$$

may also be written as

$$2y(t+1)dy = -\left(1+y^2\right)dt$$

Therefore it is also separable and we have

$$\frac{2ydy}{1+y^2} = -\frac{dt}{t+1}$$

or

$$\ln\left(1+y^2\right) = -\ln(t+1) + c$$

or

$$\left(1+y^2\right)(1+t) = k_1$$

or

$$\left(1+y^2\right)t + y^2 = k$$

1 (c) (10 pts) Solve the initial value problem

$$y'' - y' - 2y = 20e^{4t}, \;\; y(0) = 0, \;\; y'(0) = 1.$$

Solution: The characteristic equation is

$$p(r) = r^2 - r - 2 = (r-2)(r-1) = 0$$

so

$$y_h = c_1e^{2t} + c_2e^{-t}$$

Since $p(4) = 4^2 - 4 - 2 = 10 \neq 0$

$$y_p = \frac{Ke^{\alpha t}}{p(\alpha)} = \frac{20}{10}e^{4t} = 2e^{4t}$$

Thus

$$y = y_h + y_p = c_1e^{2t} + c_2e^{-t} + 2e^{4t}$$
$$y' = 2c_1e^{2t} - c_2e^{-t} + 8e^{2t}$$

The initial conditions imply

$$y(0) = c_1 + c_2 + 2 = 0$$
$$y'(0) = 2c_1 - c_2 + 8 = 1$$

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or

$$c_1 + c_2 = -2$$
$$2c_1 - c_2 = -7$$

Thus $c_1 = -3$ and $c_2 = 1$ so

$$y = -3e^{2t} + e^{-t} + 2e^{4t}$$

$$y'' - y' - 2y = 20e^{4t}$$

SNB check: $y(0) = 0$, $y'(0) = 1$, Exact solution is: $y(t) = 2e^{4t} - 3e^{2t} + e^{-t}$

2. (a) (12 pts) Find a general solution of

$$y'' + 4y = t + \cos 2t$$

Solution: The characteristic equation is $p(r) = r^2 + 4,$ so $r = \pm 2i$ and

$$y_h = c_1 \sin 2t + c_2 \cos 2t$$

The particular solution for $y'' + 4y = t$ is of the form $y_{p_1} = A_0 + A_1 t.$ Differentiating and substituting into the DE leads to

$$A_0 + 4A_1 t = t$$

so $A_0 = 0, A_1 = \frac{1}{4}$ and

$$y_{p_1} = \frac{1}{4}t$$

To find a particular solution for $\cos 2t$ we consider the equation

$$y'' + 4y = \cos 2t$$

and the companion equation

$$v'' + 4v = \sin 2t$$

Then multiplying the second equation by i and adding to the first and letting $w = y + iv$ gives

$$w'' + 4w = \cos 2t + i \sin 2t = e^{2it}$$

Since $p(r) = r^2 + 4$ and $p(2i) = 0,\ p'(r) = 2r$ so $p'(2i) = 4i \neq 0$ we have

$$w_p = \frac{te^{2it}}{4i} = -\frac{it(\cos 2t + i \sin 2t)}{4}$$

Thus

$$y_{p_2} = \operatorname{Re} w_p = \frac{1}{4} t \sin 2t$$

and a particular solution is

$$y_p = \frac{1}{4}t + \frac{1}{4} t \sin 2t$$

Thus

$$y = y_h + y_p = y_h = c_1 \sin 2t + c_2 \cos 2t + \frac{1}{4}t + \frac{1}{4} t \sin 2t$$

SNB check: $y'' + 4y = t + \cos 2t$, Exact solution is: $\frac{1}{4}t + \frac{1}{16}\cos 2t + C_1 \cos 2t - C_2 \sin 2t + \frac{1}{4} t \sin 2t,$

Alternative method to find a particular solution for $\cos 2t$: Since $\cos 2t$ is a homogeneous solution, we let

$$
\begin{aligned}
y_{p_2} &= At\sin 2t + Bt\cos 2t \\
y_{p_2}' &= A\sin 2t + 2At\cos 2t + B\cos 2t - 2Bt\sin 2t \\
y_{p_2}'' &= 4A\cos 2t - 4At\sin 2t - 4B\sin 2t - 4Bt\cos 2t
\end{aligned}
$$

The DE implies

$$4A\cos 2t - 4At\sin 2t - 4B\sin 2t - 4Bt\cos 2t + 4At\sin 2t + 4Bt\cos 2t = \cos 2t$$

so

$$
\begin{aligned}
4A - 4B &= 1 \\
B &= 0
\end{aligned}
$$

so $A = \frac{1}{4}$ and we get $y_{p_2} = \frac{1}{4}t\sin 2t$ as before.

2(b) (13 pts) Find a general solution of

$$y'' - y = \frac{1}{1+e^t}$$

Note: $\int\left(\frac{e^{-t}}{1+e^t}\right)dt = -e^{-t} - \ln(e^t) + \ln(1+e^t) + C.$

Solution: We use the Method of Variation of Parameters. The homogeneous equation has the solution

$$y_h = c_1e^t + c_2e^{-t}$$

so we let $v_1 = e^t$ and $v_2 = e^{-t}$. Then

$$
\begin{aligned}
v_1'e^t + v_2'e^{-t} &= 0 \\
v_1'e^t - v_2'e^{-t} &= \frac{1}{1+e^t}
\end{aligned}
$$

$$v_1' = \frac{\begin{vmatrix} 0 & e^{-t} \\ \frac{1}{1+e^t} & -e^{-t} \end{vmatrix}}{\begin{vmatrix} e^t & e^{-t} \\ e^t & -e^{-t} \end{vmatrix}} = -\frac{\frac{e^{-t}}{1+e^t}}{-2}$$

$$v_2' = \frac{\begin{vmatrix} e^t & 0 \\ e^t & \frac{1}{1+e^t} \end{vmatrix}}{\begin{vmatrix} e^t & e^{-t} \\ e^t & -e^{-t} \end{vmatrix}} = \frac{\frac{e^t}{1+e^t}}{-2}$$

Thus

$$v_1 = \frac{1}{2}[-e^{-t} - \ln(e^t) + \ln(1+e^t)]$$

and

$$v_2 = -\frac{1}{2}\ln(1+e^t)$$

so

$$y_p = v_1e^t + v_2e^{-t} = -\frac{1}{2} - \frac{1}{2}e^t\ln e^t + \frac{1}{2}e^t\ln(1+e^t) - \frac{1}{2}e^{-t}\ln(1+e^t)$$

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and

$$y = y_h + y_p = c_1e^t + c_2e^{-t} - \frac{1}{2} - \frac{1}{2}e^t \ln e^t + \frac{1}{2}e^t \ln(1+e^t) - \frac{1}{2}e^{-t}\ln(1+e^t)$$

$y'' - y = \frac{1}{1+e^t}$, Exact solution is: $\frac{1}{2}C_{11}e^t - \frac{1}{2}te^t + C_{12}e^{-t} + \frac{1}{2}e^t\ln(e^t+1) - \frac{1}{2e^t}\ln(e^t+1) - \frac{1}{2}$

3. (a) (10 pts) Let $\mathcal{L}\{y\} = Y(s)$Show that

$$\mathcal{L}\left[ty'\right] = -Y(s) - sY'(s)$$

Solution: Since

$$\mathcal{L}\left[ty'\right] = -\frac{d}{ds}\mathcal{L}\left[y'(t)\right] = -\frac{d}{ds}[sY(s) - y(0)] = -Y(s) - sY'(s)$$

.

(b i) (8 pts) Use Laplace transforms and the identity in 3. (a) to find a differential equation for $\mathcal{L}\{y\} = Y(s)$, where $y(t)$ is the solution of the initial value problem

$$y'' + 2ty' - 4y = 1 \quad y(0) = y'(0) = 0$$

Solution: Taking the Laplace transform of the DE implies

$$s^2\mathcal{L}\{y\} - sy(0) - y'(0) + 2\mathcal{L}\left[ty'\right] - 4\mathcal{L}\{y\} = \mathcal{L}[1] = \frac{1}{s}$$

Since

$$\mathcal{L}\left\{ty'\right\} = -Y(s) - sY'(s)$$

and the initial conditions are both equal to 0 the equation above becomes

$$s^2Y(s) - 2Y(s) - 2sY'(s) - 4Y(s) = \frac{1}{s}$$

$$\left(s^2 - 6\right)Y(s) - 2sY'(s) = \frac{1}{s}$$

Thus we have the following first order DE for $Y(s)$

$$Y'(s) + \left(\frac{3}{s} - \frac{s}{2}\right)Y = -\frac{1}{2s^2}$$

(b ii) (7 pts.) Solve the differential equation you obtained in 3 (b i) to obtain an expression for $Y(s)$. Do **not** invert your expression for $Y(s) = \mathcal{L}\{y\}$.

Solution: This is a first order linear DE for $Y(s)$ and has the integrating factor

$$e^{\int\left(\frac{3}{s}-\frac{s}{2}\right)ds} = e^{3\ln s - \frac{s^2}{4}} = s^3e^{-\frac{s^2}{4}}$$

Multiplying the DE for Y by this integrating factor yields

$$\frac{d}{ds}\left(s^3e^{-\frac{s^2}{4}}Y\right) = -\frac{s}{2}e^{-\frac{s^2}{4}}$$

so

$$s^3e^{-\frac{s^2}{4}}Y = -\int \frac{s}{2}e^{-\frac{s^2}{4}}ds = e^{-\frac{s^2}{4}} + c$$

$$Y = \frac{1}{s^3} + \frac{c}{s^3}e^{\frac{s^2}{4}}$$

4. (a) (15 pts.) Find the eigenvalues and eigenfunctions for

$$x^2y'' + xy' + \lambda y = 0 \qquad y'(1) = y'(e) = 0$$

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Be sure to consider all possible values of λ.
Solution: This is a Cauchy-Euler equation with $p = 1, q = \lambda$. The indicial equation is

$$m^2 + (1-1)m + \lambda = 0$$

or

$$m = \pm\sqrt{\lambda}$$

There are 3 cases to consider.
I. $\lambda > 0,$ say $\lambda = \alpha^2,$ where $\alpha \neq 0.$ Then

$$y = c_1 x^{\alpha} + c_2 x^{-\alpha}$$
$$y'(x) = c_1 \alpha x^{\alpha-1} - \alpha c_2 x^{-\alpha-1}$$

Hence

$$y'(1) = c_1\alpha - c_2\alpha = 0 \qquad \Rightarrow c_1 = c_2$$
$$y'(e) = c_1\alpha\left(e^{\alpha-1} - e^{-\alpha-1}\right) = 0 \quad \Rightarrow c_1 = 0$$

Hence $y = 0$ and there are no eigenvalues for this case.

II. $\lambda = 0.$ Then

$$y = c_1 + c_2 \ln x$$

and

$$y'(x) = c_2 \frac{1}{x}$$

The boundary conditions imply that $c_2 = 0,$ so $y(x) = c_1 \neq 0$ is an eigenfunction corresponding to $\lambda = 0.$

III. $\lambda < 0.$ Let $\lambda = -\beta^2$ where $\beta \neq 0.$ Then $m = \pm\beta i$ and

$$y = c_1 \cos(\beta \ln x) + c_2 \sin(\beta \ln x)$$
$$y'(x) = -c_1\left(\frac{\beta}{x}\right)\sin(\beta \ln x) + c_2\left(\frac{\beta}{x}\right)\cos(\beta \ln x)$$
$$y'(1) = \beta c_2 = 0$$

so $c_2 = 0.$

$$y'(e) = -c_1\left(\frac{\beta}{e}\right)\sin(\beta) = 0$$

Thus

$$\beta = n\pi, \quad n = 1, 2, \ldots$$

The eigenvalues are therefore

$$\lambda = -n^2\pi^2, \quad n = 1, 2, \ldots$$

with corresponding eigenfunctions

$$y_n(x) = A_n \cos(n\pi \ln x) \;\; n = 1, 2, \ldots$$

One may include the case $\lambda = 0$ by including $n = 0.$ Thus all the eigenvalues and eigenfunctions are given by

$$\lambda = -n^2\pi^2 \quad y_n(x) = A_n \cos(n\pi \ln x) \;\; n = 0, 1, 2, \ldots$$

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4(b) (10 pts.) Use separation of variables, $u(x,t) = X(x)T(t)$, to find two ordinary differential equations which $X(x)$ and $T(t)$ must satisfy to be a solution of

$$-3x^2t^4\frac{\partial u}{\partial x} + (x+1)^6(t-5)^5\frac{\partial^2 u}{\partial t^2} = 0$$

Note: Do **not** solve these ordinary differential equations.
Solution:

$$-3x^2t^4X'T + (x+1)^6(t-5)^5XT'' = 0$$

or

$$\frac{-3x^2X'}{(x+1)^6X} = -\frac{(t-5)^5T''}{t^4T} = c$$

where c is a constant. Thus

$$-3x^2X' - c(x+1)^6X = 0$$
$$(t-5)^5T'' + ct^4T = 0$$

5. (a) (15 pts.) Find the first five nonzero terms of the Fourier *cosine* series for the function

$$f(x) = \begin{cases} 0 & 0 \le x \le \frac{\pi}{2} \\ x & \frac{\pi}{2} < x \le \pi \end{cases}$$

Be sure to give the Fourier series with these terms in it. Note: $\int x\cos bx dx = \frac{1}{b^2}(\cos bx + bx\sin bx) + C$

Solution:

$$f(x) = b_0 + \sum_{n=1}^{\infty} b_n\cos\left(\frac{n\pi x}{L}\right)$$
$$b_0 = \frac{1}{L}\int_0^L f(x)dx \quad b_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx \;\; n = 1,2,\ldots$$

Here $L = \pi$ so

$$f(x) = b_0 + \sum_{n=1}^{\infty} b_n\cos(nx)$$
$$b_0 = \frac{1}{\pi}\int_0^{\pi} f(x)dx = \frac{1}{\pi}\int_{\frac{\pi}{2}}^{\pi} xdx = \frac{3}{8}\pi$$
$$b_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx = \frac{2}{\pi}\int_{\frac{\pi}{2}}^{\pi} x\cos nxdx \;\; n = 1,2,\ldots$$

so

$$b_n = \left(\frac{2}{\pi}\right)\left(\frac{1}{n^2}\right)[\cos nx + nx\sin nx]_{\frac{\pi}{2}}^{\pi} = \left(\frac{2}{n^2\pi}\right)\left[(-1)^n - \cos\left(\frac{n\pi}{2}\right) - \left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi}{2}\right)\right] \;\; n = 1,2,\ldots$$

Thus

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$$b_1 = \left(\frac{2}{\pi}\right)\left(-1 - \frac{\pi}{2}\right)$$
$$b_2 = \left(\frac{2}{4\pi}\right)[1+1] = \frac{1}{\pi}$$
$$b_3 = \left(\frac{2}{9\pi}\right)\left[-1 + \left(\frac{3\pi}{2}\right)\right]$$
$$b_4 = \left(\frac{2}{16\pi}\right)[1-1] = 0$$
$$b_5 = \left(\frac{2}{25\pi}\right)\left[-1 - \left(\frac{5\pi}{2}\right)\right]$$

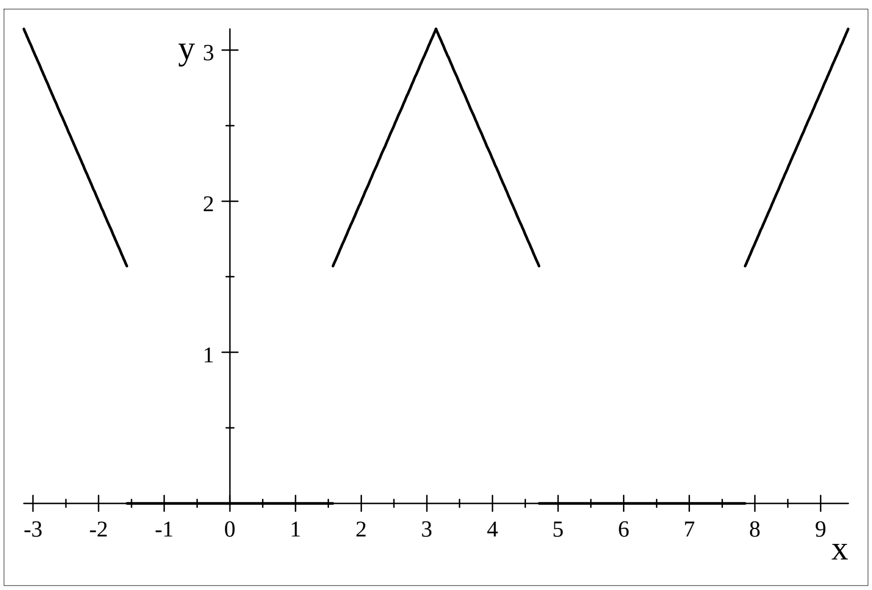
and

$$f(x) = \frac{3}{8}\pi + \left(\frac{2}{\pi}\right)\left(-1 - \frac{\pi}{2}\right)\cos x + \frac{1}{\pi}\cos 2x + \left(\frac{2}{9\pi}\right)\left[-1 + \left(\frac{3\pi}{2}\right)\right]\cos 3x + 0 \cdot \cos 4x + \left(\frac{2}{25\pi}\right)\left[-1 - \left(\frac{5\pi}{2}\right)\right]$$

(b) (10 pts.) Sketch the graph of the function represented by the Fourier cosine series in 5 (a) on $-\pi \le x \le 2\pi$.

$$f(x) = \begin{cases} 0 & 0 \le x \le \frac{\pi}{2} \\ x & \frac{\pi}{2} < x \le \pi \end{cases} \qquad \frac{\pi}{2} = 1.5708$$

x



6 (25 pts) Solve

$$\begin{array}{ll} \text{PDE} & u_{xx} = u_t \\ \text{BCs} & u(0,t) = 0 \qquad u(\pi,t) = 0 \\ \text{ICs} & u(x,0) = -17\sin 5x \end{array}$$

You must derive the solution. Your solution should not have any arbitrary constants in it. Show **all** steps.

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Solution: $u(x,t) = X(x)T(t)$

$$X''T = XT' \Rightarrow \frac{X''}{X} = \frac{T'}{T} = -\lambda^2$$

Therefore

$$X'' + \lambda^2 X = 0 \quad X(0) = X(\pi) = 0$$

$$X(x) = c_1 \sin\lambda x + c_2 \cos\lambda x$$

$X(0) = 0 \Rightarrow c_2 = 0,\; X(\pi) = c_1\sin(\lambda\pi) = 0$ so

$$\lambda = n, \quad n = 1,2,3,\ldots$$

and

$$X_n(x) = c_n \sin(nx)$$

The equation for T is

$$T' + \lambda^2 T = T' + n^2 T = 0 \Rightarrow T_n(t) = a_n e^{-n^2 t}$$

$$u_n(x,t) = X_n(x)T_n(t) = A_n \sin nx e^{-n^2 t},\; n = 1,2,\ldots$$

so

$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin nx e^{-n^2 t}$$

$$u(x,0) = \sum_{n=1}^{\infty} A_n \sin nx = -17\sin 5x$$

Therefore $A_5 = -17, A_n = 0, n \neq 5$ so

$$u(x,t) = -17\sin 5x e^{-25t}$$

7. (a) (15 pts) Find the power series solution to

$$y'' - xy' + 2y = 0$$

near $x = 0$. Be sure to give the recurrence relation. Indicate the two linearly independent solutions and give the first six nonzero terms of the solution.

Solution:

$$y(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$y'(x) = \sum_{n=1}^{\infty} a_n (n) x^{n-1}$$

$$y''(x) = \sum_{n=2}^{\infty} a_n (n)(n-1) x^{n-2}$$

The DE implies

$$\sum_{n=2}^{\infty} a_n (n)(n-1) x^{n-2} - \sum_{n=1}^{\infty} a_n (n) x^n + 2\sum_{n=0}^{\infty} a_n x^n = 0$$

or

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$$2a_0 + \sum_{n=2}^{\infty} a_n(n)(n-1)x^{n-2} + \sum_{n=1}^{\infty} a_n(2-n)x^n = 0$$

We shift the second sum by letting $k-2 = n$ or $k = n+2$ and get

$$2a_0 + \sum_{n=2}^{\infty} a_n(n)(n-1)x^{n-2} + \sum_{k=3}^{\infty} a_{k-2}(2-k+2)x^{k-2} = 0$$

Replacing n and k by m and combining we have

$$2a_0 + +(2)(1)a_2 + \sum_{m=3}^{\infty} \{a_m(m)(m-1) + a_{m-2}(4-m)\}x^{m-2} = 0$$

so

$$2a_0 + (2)(1)a_2 = 0 \Rightarrow a_2 = -a_0$$

$$a_m(m)(m-1) + a_{m-2}(4-m) = 0 \quad m = 3,4,\ldots$$

or

$$a_m = -\left(\frac{4-m}{m(m-1)}\right)a_{m-2} \quad m = 3,4,\ldots$$

Then

$$a_3 = -\frac{1}{3(2)}a_1$$

$$a_4 = 0$$

$$a_5 = \frac{1}{5(4)}a_3 = -\frac{1}{5!}a_1$$

$$a_6 = 0$$

$$a_7 = \frac{3}{7(6)}a_5 = -\frac{3}{7!}a_1$$

Thus

$$y(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \cdots = a_0\left[1 - x^2\right] + a_1\left[x - \frac{1}{3!}x^3 - \frac{1}{5!}x^5 - \frac{3}{7!}x^7 + \cdots\right]$$

SNB Check: $y'' - xy' + 2y = 0$, Series solution is: $y(x) = y(0) + xy'(0) - y(0)x^2 - \frac{1}{6}y'(0)x^3 - \frac{1}{120}y'(0)x^5 - \frac{1}{1680}y'(0)x^7$,

(b) (10 pts) Find a general solution of

$$y'' - 4y' + 4y = 3e^{2t} + 8$$

Solutions: The characteristic equation is

$$p(r) = r^2 - 4r + 4 = (r-2)^2 = 0$$

Thus $r = 2$ is a repeated root and the homogeneous solution is

$$y_h = c_1 e^{2t} + c_2 t e^{2t}$$

Since $p(2) = p'(2) = 0$ a particular solution for $3e^{2t}$ is

$$y_{p_1} = \frac{kt^2 e^{\alpha t}}{p''(\alpha)} = \frac{3}{2}t^2 e^{2t}$$

By inspection we see that a particular solution for 8 is 2. Thus

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$$y = y_h + y_{p_1} + y_{p_2} = c_1 e^{2t} + c_2 t e^{2t} + \frac{3}{2} t^2 e^{2t} + 2$$

SNB Check: $y'' - 4y' + 4y = 3e^{2t} + 8$, Exact solution is: $C_1 e^{2t} + C_2 t e^{2t} + \frac{3}{2} t^2 e^{2t} + 2$

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Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$		
$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	$n \geq 1$	$s > 0$
e^{at}	$\frac{1}{s-a}$		$s > a$
$\sin bt$	$\frac{b}{s^2 + b^2}$		$s > a$
$\cos bt$	$\frac{s}{s^2 + b^2}$		$s > a$
$e^{at}f(t)$	$\mathcal{L}\{f\}(s-a)$		
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$		

Table of Integrals

$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int x \cos bx dx = \frac{1}{b^2}(\cos bx + bx \sin bx) + C$
$\int x \sin bx = \frac{1}{b^2}(\sin bx - bx \cos bx) + C$
$\int \left(\frac{e^{-t}}{1+e^t}\right) dt = -e^{-t} - \ln(e^t) + \ln(1+e^t) + C$
$\int xe^{ax} dx = \frac{1}{a^2}(axe^{ax} - e^{ax}) + C$