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Ma 221 **Final Exam** **12/20/05**

Print Name: _________________________________ **Lecture Section**: _________

I pledge my honor that I have abided by the Stevens Honor System.

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This exam consists of 8 **problems**. **You are to solve all of these problems**. **The point value of each problem is indicated**. **The total number of points is** 200.

If you need more work space, continue the problem you are doing on the **other side of the page it is on**. Be sure that you do all problems.

You may not use a calculator, **cell phone**, **or computer while taking this exam**. **All work must be shown to obtain full credit**. **Credit will not be given for work not reasonably supported**. **When you finish**, **be sure to sign the pledge**.

There are tables giving Laplace transforms and integrals at the end of the exam.

Score on Problem #1 ________

#2 ________

#3 ________

#4 ________

#5 ________

#6 ________

#7 ________

#8 ________

Total Score _________

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1. Solve the initial value problem

(a) (8 pts)

$$\frac{dy}{dx} = \frac{3x\sec y}{\sqrt{1-x^2}}, \quad y(0) = \frac{\pi}{2}$$

(b) (7 pts) Solve

$$\left(3x^2\sin y - 5x^3\right)dx + \left(x^3\cos y + 5e^y\right)dy = 0$$

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1 (c) (10 pts) Find a general solution of

$$y'' + 10y' + 25y = 14e^{-5t}$$

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2. (a) (12 pts) Find a general solution of

$$y'' - 4y' = t - 3\sin 2t$$

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2(b) (13 pts.) Find a general solution of

$$x^2y'' + xy' - 9y = \frac{4}{x^3}$$

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3. (a) (15 pts.) Find:

$$\mathcal{L}^{-1}\left\{\frac{s+3}{(s^2+1)(s^2-1)}\right\}$$

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(b) (10 pts.) Solve using Laplace Transforms:

$$y'' + 2y' + 5y = 0 \qquad y(0) = 1, \;\; y'(0) = 2$$

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4.) Assume that the problem

$$\text{PDE}: \quad u_{tt} = u_{xx} + 2u_x; \quad 0 < x < \pi, \; t > 0$$

$$\text{B.C.}: \quad u(0,t) = 0, \quad u(\pi,t) = 0$$

has a solution of the form $u(x,t) = X(x)T(t)$.

a.) (10 pts.) Use separation of variables to derive the following eigenvalue problem for $X(x)$:

$$X'' + 2X' + \lambda X = 0; \quad X(0) = 0, \quad X(\pi) = 0$$

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b.) (15 pts.) Find the eigenvalues λ and the corresponding eigenfunctions for the problem

$$X'' + 2X' + \lambda X = 0; \quad X(0) = 0, \quad X(\pi) = 0$$

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5. (a) (15 pts.) Find the first five nonzero terms of the Fourier *sine* series for the function

$$f(x) = \begin{cases} -2 & 0 \le x \le \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < x \le \pi \end{cases}$$

Be sure to give the Fourier series with these terms in it.

(b) (10 pts.) Sketch the graph of the function represented by the Fourier sine series in 5 (a) on $-\pi \le x \le 3\pi$.

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6 (25 pts.)

$$\text{PDE} \qquad u_{xx} = 9u_{tt}$$
$$\text{BCs} \quad u(0,t) = 0 \qquad u_x(1,t) = 0$$
$$\text{ICs} \quad u(x,0) = 0 \quad u_t(x,0) = \frac{\pi}{6}\sin\left(\frac{7\pi x}{2}\right)$$

You must derive the solution. Your solution should not have any arbitrary constants in it. Show **all** steps.

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7. (a) (15 pts.) Find the power series solution to

$$y'' - x^2 y = 0$$

near $x = 0$. Be sure to give the recurrence relation. Indicate the two linearly independent solutions and give the first six nonzero terms of the solution.

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(b) (10 pts.) Solve

$$y' + y = ty^3 \quad y(0) = 2$$

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8 (a) (15 pts.)The functions $y_1(t) = t$ and $y_2(t) = e^t$ are known to be solutions of

$$y'' + p(t)y' + q(t)y = 0$$

Determine the two functions $p(t)$ and $q(t)$.

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(b) (10 pts.) Let $\{\Phi_n(x)\}$, $n = 1, 2, \ldots$ be a set of orthonormal functions on $[a. b]$, that is,

$$< \Phi_n, \Phi_m >= \int_a^b \Phi_n(x)\Phi_m(x)dx = \begin{cases} 0 \text{ for } n \neq m \\ 1 \text{ for } n = m \end{cases}$$

Let

$$f(x) = \sum_{n=1}^{\infty} a_n \Phi_n(x)$$

Show that

$$\sum_{i=1}^{\infty} [a_i]^2 = \int_a^b [f(x)]^2 dx$$

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Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$		
$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	$n \geq 1$	$s > 0$
e^{at}	$\frac{1}{s-a}$		$s > a$
$\sin bt$	$\frac{b}{s^2 + b^2}$		$s > a$
$\cos bt$	$\frac{s}{s^2 + b^2}$		$s > a$
$e^{at}f(t)$	$\mathcal{L}\{f\}(s-a)$		
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$		

Table of Integrals

$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int x \cos bx dx = \frac{1}{b^2}(\cos bx + bx \sin bx) + C$
$\int x \sin bx = \frac{1}{b^2}(\sin bx - bx \cos bx) + C$
$\int \left(\frac{e^{-t}}{1+e^t}\right) dt = -e^{-t} - \ln(e^t) + \ln(1+e^t) + C$
$\int xe^{ax} dx = \frac{1}{a^2}(axe^{ax} - e^{ax}) + C$