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Ma 221 **Final Exam** **5/8/07**

Print Name: _________________________________

Lecture Section: _________

I pledge my honor that I have abided by the Stevens Honor System.

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This exam consists of 8 problems. You are to solve all of these problems. The point value of each problem is indicated. The total number of points is 200.

If you need more work space, continue the problem you are doing on the **other side of the page it is on**. Be sure that you do all problems.

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

There are tables giving Laplace transforms and integrals at the end of the exam.

Score on Problem #1 ________

#2 ________

#3 ________

#4 ________

#5 ________

#6 ________

#7 ________

#8 ________

Total Score ________

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1. Solve

(a) (8 pts)

$$\left(2y^2x - 2y^3\right)dx + \left(4y^3 - 6y^2x + 2yx^2\right)dy = 0$$

(b) (7 pts) Solve

$$xy' - 2y = \frac{2}{3}x^5 \quad y(1) = \frac{2}{9}$$

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1 (c) (10 pts) Find a general solution of

$$y'' - y' - 2y = e^{-5t} + 3e^{2t}$$

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2. (a) (12 pts) Find a general solution of

$$y'' + 2y' - 3y = 5\sin 3t - 3 + 3t^2$$

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2(b) (13 pts.) Find a general solution of

$$y'' - 2y' + y = \frac{e^x}{x}$$

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3. (a) (10 pts.) Let

$$g(t) = \begin{cases} 1 & \text{for } 0 \le t \le 1 \\ 0 & \text{for } t \ge 1 \end{cases}$$

Use the definition of the Laplace transform to find $\mathcal{L}\{g(t)\}$

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(b) (15 pts.) Solve using Laplace Transforms:

$$y'' + 4y = 4x \qquad y(0) = 1,\ \ y'(0) = 5$$

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4.) a.) (10 pts.) Use separation of variables, $u(x,t) = X(x)T(t)$, to find two ordinary differential equations which $X(x)$ and $T(t)$ must satisfy to be a solution of

$$-12x^2t^5\frac{\partial^2 u}{\partial t^2} + (x+2)^3(t+2)^5\frac{\partial u}{\partial x} = 0.$$

Note: Do **not** solve these ordinary differential equations.

b.) (15 pts.) Find the eigenvalues and eigenfunctions of

$$y'' - 4\lambda y' + 4\lambda^2 y = 0 \quad y(0) = 0 \quad y(1) + y'(1) = 0$$

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5. (a) (15 pts.) Find the first five nonzero terms of the Fourier *sine* series for the function

$$f(x) = \begin{cases} 2 & 0 \le x \le \frac{\pi}{4} \\ 0 & \frac{\pi}{4} < x \le \frac{\pi}{2} \end{cases}$$

Be sure to give the Fourier series with these terms in it.

(b) (10 pts.) Sketch the graph of the function represented by the Fourier sine series in 5 (a) on $-\pi \le x \le \pi$.

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6 (25 pts.)

$$\text{PDE} \qquad u_{xx} - 8u_t = 0$$

$$\text{BCs} \quad u(0,t) = 0 \qquad u_x(1,t) = 0$$

$$\text{ICs} \quad u(x,0) = -2\sin\frac{3\pi}{2}x + 10\sin\frac{9\pi}{2}x$$

You must derive the solution. Your solution should not have any arbitrary constants in it. Show **all** steps.

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7. (a) (15 pts.) Solve the equation

$$y'' + 3xy' + 2y = 0$$

near $x = 0$. Be sure to give the recurrence relation and the first 3 nonzero terms in each linearly independent solution.

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(b) (10 pts.) Find a second order homogeneous, ordinary differential equation with real constant coefficients for which

$$-5.2e^{-3t}\cos 5t \text{ and } \frac{2}{3}e^{-3t}\sin 5t$$

are the solutions.

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8 (a) (12 pts.) Find an integrating factor to make

$$y^2 \cos x dx + (4 + 5y \sin x) dy = 0$$

exact. Then use it to solve the equation.

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(b) (13 pts.) Find

$$\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+4s+13}\right\}$$

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Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$		
$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	$n \geq 1$	$s > 0$
e^{at}	$\frac{1}{s-a}$		$s > a$
$\sin bt$	$\frac{b}{s^2 + b^2}$		$s > 0$
$\cos bt$	$\frac{s}{s^2 + b^2}$		$s > 0$
$e^{at}f(t)$	$\mathcal{L}\{f\}(s-a)$		
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$		

Table of Integrals

$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int x \cos bx dx = \frac{1}{b^2}(\cos bx + bx \sin bx) + C$
$\int x \sin bx = \frac{1}{b^2}(\sin bx - bx \cos bx) + C$
$\int \left(\frac{e^{-t}}{1+e^t}\right) dt = -e^{-t} - \ln(e^t) + \ln(1+e^t) + C$
$\int x e^{ax} dx = \frac{1}{a^2}(axe^{ax} - e^{ax}) + C$