Name_________________________ Lecturer ______________________

Ma 221 Final Exam 5/5/08

Print Name: ________________________________

Lecture Section: _________

I pledge my honor that I have abided by the Stevens Honor System.

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This exam consists of 8 problems. You are to solve all of these problems. The point value of each problem is indicated. The total number of points is 200.

If you need more work space, continue the problem you are doing on the **other side of the page it is on**. Be sure that you do all problems.

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

There are tables giving Laplace transforms and integrals at the end of the exam.

Score on Problem #1 ________

#2 ________

#3 ________

#4 ________

#5 ________

#6 ________

#7 ________

#8 ________

Total Score ________

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1. Solve

(a) (8 pts)

$$\left(x^2+1\right)y' + 3xy = 6x \quad y(0) = -1$$

(b) (7 pts) Solve

$$\sin(x+y)dx + (2y + \sin(x+y))dy = 0$$

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1 (c) (10 pts) Find a general solution of

$$y'' + 2y' - 3y = 3t + 4 + e^{-3t}$$

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2. (a) (12 pts) Find a general solution of

$$y'' + 4y = \cos 2t - 2\sin t$$

2(b) (13 pts.) Use the method of Variation of Parameters to find a general solution of

$$y'' + 2y' + y = t^5 e^{-t}$$

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3. (a) (10 pts.) Let

$$g(t) = \begin{cases} t & \text{for } 0 \le t \le 1 \\ 1 & \text{for } 1 < t < \infty \end{cases}$$

Use the definition of the Laplace transform to find $\mathcal{L}\{g(t)\}$

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(b) (15 pts.) Solve using Laplace Transforms:

$$y'' - y' - 2y = e^{2t} \qquad y(0) = 0, \;\; y'(0) = -1$$

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4.) a.) (10 pts.) Use separation of variables, $u(x,t) = X(x)T(t)$, to find two ordinary differential equations which $X(x)$ and $T(t)$ must satisfy to be a solution of

$$-3x^4t^3\frac{\partial^2 u}{\partial x^2} + (x+6)^5(t-2)^3\frac{\partial u}{\partial t} = 0.$$

Note: Do **not** solve these ordinary differential equations.

b.) (15 pts.) Find the eigenvalues and eigenfunctions for

$$x^2y'' + xy' + \lambda y = 0 \qquad y(1) = y(e^{\pi}) = 0$$

Be sure to consider the cases $\lambda < 0, \lambda = 0,$ and $\lambda > 0.$

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5. (a) (15 pts.) Find the first five nonzero terms of the Fourier *cosine* series for the function

$$f(x) = 2 - x; \quad 0 < x < 2$$

Be sure to give the Fourier series with these terms in it.

(b) (10 pts.) Sketch the graph of the function represented by the Fourier cosine series in 5 (a) on $-2 < x < 6$.

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6 (25 pts.)

$$\begin{array}{lrcl} \text{PDE} & u_{tt} & = & 4u_{xx} \\ \text{BCs} & u_x(0,t) & = & 0 \qquad u_x(\pi,t) = 0 \\ \text{ICs} & u(x,0) & = & 2\cos(3x) - 5\cos(5x) + 7\cos(6x) \\ & u_t(x,0) & = & 0 \end{array}$$

You must derive the solution. Your solution should not have any arbitrary constants in it. Show **all** steps.

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7. (a) (10 pts.) Consider the equation

$$(x^2 - 8x + 15)y'' + (x + 2)y' + 3y = 0$$

i.) Find the singular points of the above equation

ii.) Find a **minimum value** for the radius of convergence of a power series solution about $x = 2$ of the above equation. (Note: You do NOT have to find the power series solution.)

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(b) (15 pts.) Find the first 6 nonzero terms of each linearly independent power series solution about $x = 0$ for the DE:

$$y'' - x^3 y' + y = 0$$

Be sure to give the recurrence relation.

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8 (a) (12 pts.) Consider:

$$y'' + p(t)y' + q(t)y = 1$$

Given that $y_1(t) = t$ and $y_2(t) = t^2$ are solutions to the above equation, find $p(t)$ and $q(t)$.

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(b) (13 pts.) Find

$$\mathcal{L}^{-1}\left\{\frac{5s^2-5s-4}{(s-3)(s^2+4)}\right\}$$

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Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$		
$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	$n \geq 1$	$s > 0$
e^{at}	$\frac{1}{s-a}$		$s > a$
$\sin bt$	$\frac{b}{s^2 + b^2}$		$s > 0$
$\cos bt$	$\frac{s}{s^2 + b^2}$		$s > 0$
$e^{at} f(t)$	$\mathcal{L}\{f\}(s-a)$		
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$		

Table of Integrals

$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int x \cos bx dx = \frac{1}{b^2}(\cos bx + bx \sin bx) + C$
$\int x \sin bx = \frac{1}{b^2}(\sin bx - bx \cos bx) + C$
$\int \left(\frac{e^{-t}}{1+e^t}\right) dt = -e^{-t} - \ln(e^t) + \ln(1+e^t) + C$
$\int xe^{ax} dx = \frac{1}{a^2}(axe^{ax} - e^{ax}) + C$