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Ma 221 **Final Exam** **5/15/12**

Print Name: _________________________________

Lecture Section: _________

I pledge my honor that I have abided by the Stevens Honor System.

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This exam consists of 8 problems. You are to solve all of these problems. The point value of each problem is indicated. The total number of points is 200.

If you need more work space, continue the problem you are doing on the **other side of the page it is on**. Be sure that you do all problems.

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

There are tables giving Laplace transforms and integrals at the end of the exam.

Score on Problem #1a ________

#1b________

#1c________

#2a ________

#2b________

#3a ________

#3b_________

#4a ________

#4b_________

#5a ________

#5b_________

#6 ________

#7a ________

#7b________

#8a ________

#8b________

Total Score ________

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1. Solve

(a) (8 pts)

$$2xy^3dx - (1 - y^2)dy = 0 \quad y(1) = 1$$

(b) (7 pts) Solve

$$(8x^3y^3 + 4x^3)dx + (6x^4y^2 + 4y^3)dy = 0$$

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1 (c) (10 pts) Find a general solution of

$$y'' + 2y' - 3y = 8e^t + 18t$$

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2. (a) (12 pts) Find a general solution of

$$y'' + 4y' + 5y = 16\sin t$$

2(b) (13 pts.) Find a general solution of

$$y''(\theta) + 16y(\theta) = \tan 4\theta$$

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3. (a) (10 pts.) Use the definition of the Laplace transform to find $\mathscr{L}\{f(t)\}$ where

$$f(t) = \begin{cases} e^{2t} & 0 \leq t < 3 \\ 1 & 3 < t \end{cases}$$

for $s > 2$.

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(b) (15 pts.) Solve using Laplace Transforms:

$$y'' + 2y' + y = t^2 + 4t \qquad y(0) = 0, \;\; y'(0) = -1$$

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4.) a.) (10 pts.) Use separation of variables, $u(x,t) = X(x)T(t)$, to find two ordinary differential equations which $X(x)$ and $T(t)$ must satisfy to be a solution of

$$\left(t^2+1\right)u_{xx} + x^2\left(t^2+1\right)u_x - \left(x^2+1\right)u_{tt} = 0$$

Note: Do **not** solve these ordinary differential equations.

b.) (15 pts.) Find all eigenvalues (λ) and the corresponding eigenfunctions for the boundary value problem

$$y'' + 2y - \lambda y = 0 \qquad y'(0) = y'(\pi) = 0$$

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5. (a) (15 pts.) Find the Fourier *sine* series for the function

$$f(x) = x \quad \text{on } 0 < x < 1$$

(b) (10 pts.) Sketch the graph of the function represented by the Fourier *sine* series in 5 (a) on $-3 < x < 3$.

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6 (25 pts.)

$$\text{PDE} \quad u_{xx} - 16u_{tt} = 0$$
$$\text{BCs} \quad u(0,t) = 0 \qquad u_x(1,t) = 0$$
$$\text{IC} \quad u(x,0) = -6\sin\left(\frac{3\pi x}{2}\right) + 13\sin\left(\frac{11\pi x}{2}\right)$$
$$\text{IC} \quad u_t(x,0) = 0$$

You must derive the solution. Your solution should not have any arbitrary constants in it. Show **all** steps.

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7. (a) (10 pts.) Solve

$$x^2 y'' + 3xy' + 2y = 0$$

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(b) (15 pts.) Find the first 5 nonzero terms of the power series solution about $x = 0$ for the DE:

$$y'' - 4xy' + 4y = 0$$

Be sure to give the recurrence relation.

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8 (a) (10 pts.) Solve

$$\frac{dy}{dx} + y = \frac{e^x}{y} \quad y(0) = 1$$

(b) (15 pts.) Find

$$\mathcal{L}^{-1}\left\{\frac{25}{s^3(s^2+4s+5)}\right\}$$

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Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$		
$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	$n \geq 1$	$s > 0$
e^{at}	$\frac{1}{s-a}$		$s > a$
$\sin bt$	$\frac{b}{s^2 + b^2}$		$s > 0$
$\cos bt$	$\frac{s}{s^2 + b^2}$		$s > 0$
$e^{at}f(t)$	$\mathcal{L}\{f\}(s-a)$		
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$		

Table of Integrals

$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int x \cos bx dx = \frac{1}{b^2}(\cos bx + bx \sin bx) + C$
$\int x \sin bx dx = \frac{1}{b^2}(\sin bx - bx \cos bx) + C$
$\int \sec x dx = \ln\lvert\sec x + \tan x\rvert + C$
$\int \left(\frac{e^{-t}}{1+e^{-t}}\right) dt = -\ln(1 + e^{-t}) + C$
$\int \left(\frac{e^{-2t}}{1+e^{-t}}\right) dt = \ln(1 + e^{-t}) - e^{-t} + C$
$\int x e^{ax} dx = \frac{1}{a^2}(axe^{ax} - e^{ax}) + C$