Name_________________________ Lecturer ______________________

Ma 221 **Final Exam** **5/14/13**

Print Name: ________________________________

Lecture Section: _________

I pledge my honor that I have abided by the Stevens Honor System.

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This exam consists of 8 problems. You are to solve all of these problems. The point value of each problem is indicated. The total number of points is 200.

If you need more work space, continue the problem you are doing on the **other side of the page it is on**. Be sure that you do all problems.

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

There are tables giving Laplace transforms and integrals at the end of the exam.

Score on Problem #1a ________

#1b ________

#1c ________

#2a ________

#2b ________

#3a ________

#3b ________

#4a ________

#4b ________

#5a ________

#5b ________

#6 ________

#7a ________

#7b ________

#8a ________

#8b ________

Total Score ________

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1. Solve

(a) (8 pts)

$$\frac{dy}{dx} = \frac{e^{x+y}}{y-1} \qquad y(0) = 0$$

(b) (7 pts) Solve

$$y\frac{dx}{dy} + 2x = 5y^3$$

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1 (c) (10 pts) Solve

$$\frac{dy}{dx} = \frac{x}{y} + \frac{y}{x}, \qquad y(1) = 4$$

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2. (a) (12 pts) Find a general solution of

$$y'' + 4y' + 5y = 10 + 8\sin t$$

2(b) (13 pts.) Find a general solution of

$$y'' + 16y = \tan 4t$$

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3. (a) (10 pts.) Let $F(s) = \mathcal{L}\{f(t)\}$. Given that $f(t)$ is continuous and of exponential order α, use the definition of the Laplace Transform to show that

$$\mathcal{L}\{tf(t)\} = -\frac{dF}{ds}.$$

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(b) (15 pts.) Solve using Laplace Transforms:

$$y'' - 4y' + 4y = 3te^{2t} \qquad y(0) = 4, \; y'(0) = 2$$

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4.) a.) (10 pts.) Use separation of variables, $u(x,t) = X(x)T(t)$, to find two ordinary differential equations which $X(x)$ and $T(t)$ must satisfy to be a solution of

$$e^{2x+3t}\frac{\partial^2 u}{\partial x \partial t} - (x+4)^5(t^2+7)^8\frac{\partial^2 u}{\partial x^2} = 0$$

Note: Do **not** solve these ordinary differential equations.

b.) (15 pts.) Find

$$\mathcal{L}^{-1}\left\{\frac{4s^2+13s+19}{(s-1)\left(s^2+4s+13\right)}\right\}$$

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5. (a) (15 pts.) Find the first five non-zero terms of the Fourier *cosine* series for the function

$$f(x) = x \text{ on } 0 < x < \pi$$

5(b) (10 pts.) Sketch the graph of the function represented by the Fourier *cosine* series in 5 (a) on $-\pi < x < 3\pi$.

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6 (25 pts.) Solve

$$\begin{array}{ll} \text{PDE} & u_{xx} = u_t \\ \text{BCs} & u(0,t) = 0 \qquad u(\pi,t) = 0 \\ \text{ICs} & u(x,0) = -17\sin 5x \end{array}$$

You must derive the solution. Your solution should not have any arbitrary constants in it. Show **all** steps.

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7. (a) (10 pts.) Find a general solution of

$$y'' - 4y' + 4y = 3e^{2t} + 8$$

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7 (b) (15 pts.) Find the power series solution to

$$y'' - xy' + 2y = 0$$

near $x = 0$. Be sure to give the recurrence relation. Indicate the two linearly independent solutions and give the first six nonzero terms of the solution.

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8 (a) (15 pts.) Find the eigenvalues and eigenfunctions for

$$x^2y'' + xy' + \lambda y = 0 \qquad y(1) = y(e^\pi) = 0$$

Be sure to consider the cases $\lambda < 0, \lambda = 0,$ and $\lambda > 0.$

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8(b) (10 pts.) Show that if the equation

$$\left(\frac{y^2}{2} + 2ye^x\right)dx + (y + e^x)dy = 0$$

is multiplied by e^x, then the resulting equation is exact and then solve this equation.

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Table of Laplace Transforms

$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$		
$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	$n \geq 1$	$s > 0$
e^{at}	$\frac{1}{s-a}$		$s > a$
$\sin bt$	$\frac{b}{s^2 + b^2}$		$s > 0$
$\cos bt$	$\frac{s}{s^2 + b^2}$		$s > 0$
$e^{at}f(t)$	$\mathcal{L}\{f\}(s-a)$		
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}(\mathcal{L}\{f\}(s))$		

Table of Integrals

$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int x \cos bx dx = \frac{1}{b^2}(\cos bx + bx \sin bx) + C$
$\int \tan ax dx = -\frac{1}{a}\ln(\cos ax) + C$
$\int \sec ax dx = \frac{1}{a}\ln\lvert\sec ax + \tan ax\rvert + c$
$\int \sec ax \tan ax dx = \frac{1}{a}\sec ax + C$
$\int x \sin bx dx = \frac{1}{b^2}(\sin bx - bx \cos bx) + C$
$\int \left(\frac{e^{-t}}{1+e^{-t}}\right) dt = -\ln(1+e^{-t}) + C$
$\int \left(\frac{e^{-2t}}{1+e^{-t}}\right) dt = \ln(1+e^{-t}) - e^{-t} + C$
$\int xe^{ax} dx = \frac{1}{a^2}(axe^{ax} - e^{ax}) + C$