Ma 227 – Fall 2013 Exam I review

Double integrals

- Limits of iterated integral from region of integration
- Region of integration from limits of integration
- Interchange of order of integration
- Equivalent iterated integrals in rectangular and polar coordinates

$$\iint_R f(x,y)dA = \iint_R f(x,y)dydx = \iint_R f(x,y)dxdy = \iint_R f(r\cos\theta, r\sin\theta)rdrd\theta$$

Triple integrals

- Limits of iterated integral from region of integration
- Region of integration from limits of integration
- Interchange of order of integration
- Equivalent iterated integrals in rectangular, cylindrical and spherical coordinates

$$\begin{aligned}\iiint_R f(x,y,z)dV &= \iiint_R f(x,y,z)dzdydx = \dots \\ &= \iiint_R f(r\cos\theta, r\sin\theta, z)rdzdrd\theta \\ &= \iiint_R f(\rho\cos\theta\sin\phi, \rho\sin\theta\sin\phi, \rho\cos\phi)\rho^2\sin\phi d\rho d\phi d\theta\end{aligned}$$

Application of double integral to surface area

- Graph of a function, $z = f(x,y)$

$$A(S) = \iint_R \sqrt{1 + f_x^2 + f_y^2}\, dA_{xy}$$

- Parametric representation

$$\begin{aligned}\mathbf{r}(u,v) &= x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k} \\ A(s) &= \iint_D |\mathbf{r}_u \times \mathbf{r}_v| dA_{uv}\end{aligned}$$