Ma 227 – Fall 2013 - Exam III Review

Surface Integrals

Surface integral of scalar function $\iint_S f dS = \iint_S f ds$

Surface given explicitly by $z = g(x,y)$: $dS = \sqrt{1 + g_x^2 + g_y^2}\, dA_{xy}$
and the region of integration is the domain of $g(x,y)$.

Surface given parametrically

$$\mathbf{r} = \langle x,y,z \rangle = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = \langle x(u,v), y(u,v), z(u,v) \rangle$$

$dS = |\mathbf{r}_u \times \mathbf{r}_v| dA_{uv}$ and the region of integration is the set of values of the parameters u and v which define the surface.

Surface integral of vector function $\iint_S \mathbf{F} \bullet d\mathbf{S} = \iint_S \mathbf{F} \bullet \mathbf{n} dS = \iint_D \mathbf{F} \bullet (\mathbf{r}_u \times \mathbf{r}_v) dA_{uv}$

(Here we use the parametric representation and observe that $\mathbf{r}_u \times \mathbf{r}_v$ is normal to the surface. The specified direction of the normal must be used and the opposite direction is $\mathbf{r}_v \times \mathbf{r}_u = -\mathbf{r}_u \times \mathbf{r}_v$)

Equivalencies

Stokes's Theorem (in 3D space)

$\iint_S curl\mathbf{F} \bullet d\mathbf{S} = \iint_S (\nabla \times \mathbf{F}) \bullet \mathbf{n} dS = \int_{\partial S} \mathbf{F} \bullet d\mathbf{r}$ where the direction around the boundary and of the normal must be matched.

Divergence Theorem (in 3D space)

$\iiint_E \left(\nabla \bullet \mathbf{F}\right) dV = \iint_{\partial E} \mathbf{F} \bullet d\mathbf{S} = \iint_{\partial E} \mathbf{F} \bullet \mathbf{n} dS$ and the direction of the normal must be outward from the solid.

Linear Equations and Matrices

Systems of linear equations
    row reduction
Matrix algebra
    Addition
    Multiplication by scalar
    Multiplication of matrices
    Inverse
    Inverse by row operations
    Eigenvalues and eigenvectors

$$A\mathbf{u} = r\mathbf{u}$$

Characteristic equation

$$|A - rI| = \det(A - rI) = 0$$