Ma 227 – Fall 2013

Texts: R. Kent Nagle, Edward B. Saff and Arthur David Snider: *Fundamentals of Differential Equations and Boundary Value Problems, Sixth Ed.*
James Stewart: *Calculus – Concepts and Contexts, Fourth Ed.*

Multiple integrals (Stewart, Chap. 12)

Double integrals

- Limits of iterated integral from region of integration
- Region of integration from limits of integration
- Interchange of order of integration
- Equivalent iterated integrals in rectangular and polar coordinates

$$\iint_R f(x,y)dA = \iint_R f(x,y)dydx = \iint_R f(x,y)dxdy = \iint_R f(r\cos\theta, r\sin\theta)rdrd\theta$$

Triple integrals

- Limits of iterated integral from region of integration
- Region of integration from limits of integration
- Interchange of order of integration
- Equivalent iterated integrals in rectangular, cylindrical and spherical coordinates

$$\iiint_R f(x,y,z)dV = \iiint_R f(x,y,z)dzdydx = \ldots$$

$$= \iiint_R f(r\cos\theta, r\sin\theta, z)rdzdrd\theta$$

$$= \iiint_R f(\rho\cos\theta\sin\phi, \rho\sin\theta\sin\phi, \rho\cos\phi)\rho^2\sin\phi d\rho d\phi d\theta$$

Application of double integral to surface area

- Graph of a function, $z = f(x,y)$

$$A(S) = \iint_R \sqrt{1 + f_x^2 + f_y^2}\, dA_{xy}$$

- Parametric representation

$$\mathbf{r}(u,v) = x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k}$$

$$A(s) = \iint_D |\mathbf{r}_u \times \mathbf{r}_v| dA_{uv}$$

Vector calculus (Stewart, Chap. 13)

Operations relating to vector functions (fields) $[\mathbf{F} = P\mathbf{i} + Q\mathbf{j} + Q\mathbf{k}]$

Gradient $\operatorname{grad} f = \mathbf{F} = \nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$ (input scalar, output vector)

Divergence $\operatorname{div}\mathbf{F} = \nabla \bullet \mathbf{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$ (input vector, output scalar)

Curl $\operatorname{curl}\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$ (input vector, output vector)

Line integral

Line integral with respect to arc length $\int_C \mathbf{f}\, \mathbf{ds}$

Parametrize C

Set up and evaluate in 2 or 3 dimensions

$$\int_C f\, ds = \int_a^b f(\mathbf{r}(t))|\mathbf{r}'(t)|dt$$

$$\int_C f\, ds = \int_a^b f(x(t), y(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\int_C f\, ds = \int_a^b f(x(t), y(t), z(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$

Line integral of vector function

$$\int_C \mathbf{F} \bullet d\mathbf{r} = \int_C P dx + Q dy + R dz$$

Parametrize C

Set up and evaluate in 2 or 3 dimensions

Independence of path: $\mathbf{F} = \nabla\phi$

Test: $curl\mathbf{F} = \nabla \times \mathbf{F} = \mathbf{0}$

Determination of ϕ

Surface integral of scalar function

$$\iint_S f dS = \iint_S f ds$$

Surface given explicitly by $z = g(x,y)$: $dS = \sqrt{1 + g_x^2 + g_y^2}\, dA_{xy}$
and the region of integration is the domain of $g(x,y)$.

Surface given parametrically

$$\mathbf{r} = \langle x, y, z \rangle = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = \langle x(u,v), y(u,v), z(u,v) \rangle$$

$dS = |\mathbf{r}_u \times \mathbf{r}_v| dA_{uv}$ and the region of integration is the set of values of the parameters u and v which define the surface.

Surface integral of vector function $\iint_S \mathbf{F} \bullet d\mathbf{S} = \iint_S \mathbf{F} \bullet \mathbf{n} dS = \iint_D \mathbf{F} \bullet (\mathbf{r}_u \times \mathbf{r}_v) dA_{uv}$

Here we use the parametric representation and that $\mathbf{r}_u \times \mathbf{r}_v$ is normal to the surface.

Equivalencies

Green's Theorem (in the plane)

$\iint_S \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \oint_{\partial S} P dx + Q dy$ with the direction around the boundary so that the surface is to the left.

Stokes's Theorem (in 3D space)

$\iint_S curl\mathbf{F} \bullet d\mathbf{S} = \iint_S (\nabla \times \mathbf{F}) \bullet \mathbf{n} dS = \int_{\partial S} \mathbf{F} \bullet d\mathbf{r}$ where the direction around the boundary and of the normal must be matched.

Divergence Theorem (in 3D space)

$\iiint_E \left(\nabla \bullet \mathbf{F} \right) dV = \iint_{\partial E} \mathbf{F} \bullet d\mathbf{S} = \iint_{\partial E} \mathbf{F} \bullet \mathbf{n} dS$ and the direction of the normal must be outward from the solid.

Matrices and Systems of Linear differential equations (Nagle, Chap. 9)

Systems of linear equations
- row reduction

Matrix algebra
- Addition
- Multiplication by scalar
- Multiplication of matrices
- Inverse
- Inverse by row operations

Calculus of matrices

Linear systems of differential equations
- Normal form
- Conversion of nth order d.e. to linear system
- Representation of solutions
  - Homogeneous equation
    - Fundamental solutions set and fundamental solution matrix

$$x_h = c_1\mathbf{x}_1(t) + c_2\mathbf{x}_2(t) + \ldots + c_n\mathbf{x}_n(t) = \mathbf{X}(t)\mathbf{c}$$

  - Non-homogeneous equation

$$\mathbf{x}(t) = \mathbf{x}_h(t) + \mathbf{x}_p(t)$$

- Solutions for d.e. with constant coefficients
  - Homogeneous d.e.
    - Eigenvalues and eigenvectors

$$A\mathbf{u} = r\mathbf{u}$$

      - Characteristic equation

$$|A - ri| = \det(a - rI) = 0$$

      - Linear independence of eigenvectors
      - Form of solution(s) in real & complex cases
  - Non-homogeneous d.e.
    - Undetermined coefficients
  - Matrix exponential function (omitted from course)

General review - Sunday, 15 December 2013 - K 228

1:00 - 1:50 Systems of DEs (Prof. Levine)
2:00 - 2:50 Gradient, Divergence, Curl, Line Integrals & Green’s Theorem (Prof. Levine)
3:00 - 3:50 Multiple Integration(Prof. Brady)
4:00 - 4:50 Surface Integrals, Stokes’s Theorem, Divergence Theorem (Prof. Brady)