Ma 227 – Fall 2013 - Exam II Review

Operations relating to vector functions (fields)

Gradient $\mathbf{F} = \nabla f$ (input scalar, output vector)

Divergence $\nabla \bullet \mathbf{F}$ (input vector, output scalar)

Curl $\nabla \times \mathbf{F}$ (input vector, output vector)

Line integral with respect to arc length $\int_C \mathbf{f}\, \mathbf{ds}$

Parametrize C

Set up and evaluate in 2 or 3 dimensions

$$\int_C f\, ds = \int_a^b f(\mathbf{r}(t))|\mathbf{r}'(t)|dt$$

$$\int_C f\, ds = \int_a^b f(x(t), y(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\int_C f\, ds = \int_a^b f(x(t), y(t), z(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$

Line integral $\int_C \mathbf{F} \bullet d\mathbf{r}$

Parametrize C

Set up and evaluate in 2 or 3 dimensions

$$\int_C \mathbf{F} \bullet d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \bullet \frac{d\mathbf{r}}{dt}\, dt$$

Independence of path: $\mathbf{F} = \nabla\phi$

Test: $curl\mathbf{F} = \nabla \times \mathbf{F} = \mathbf{0}$

Determination of ϕ

Green's Theorem (in the plane)

$\iint\limits_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \oint\limits_{\partial D} P dx + Q dy$ with the direction around the boundary chosen so that the region D is to the left as the boundary is traversed.

Corollary - Area of a region D in the plane

$$Area(D) = \frac{1}{2} \oint\limits_{\partial D} x dy - y dx$$