Selected problems

2009, 2010 & 2011 Ma 227 Final Exams

Surface Integrals, Stokes's Theorem, Divergence Theorem

2011 Problem 3

Evaluate the surface integral

$$\iint_S \vec{F} \bullet d\vec{S} = \iint_S \vec{F} \bullet \vec{n} dS$$

where S is the surface of the portion of the cone $z^2 = x^2 + y^2$ in the first octant and below the plane $z = 4$ with downward (outward) normal and

$$\vec{F} = yz\vec{i} + xz\vec{j} + xy\vec{k}.$$

2010 problem 4b

Evaluate the surface integral $\iint_S \vec{F} \bullet d\vec{S} = \iint_S \vec{F} \bullet \vec{n} dS$ for $\vec{F} = z\vec{i} + x\vec{j} + y\vec{k}$ with S the triangle with vertices $(1,0,0), (0,2,0)$ and $(0,0,3)$ oriented upward.

2009 problem 4

a) (**10 points**)

Evaluate

$$\iint_S \vec{F} \bullet d\,\vec{S} = \iint_S \vec{F} \bullet \vec{n} dS$$

where

$$\vec{F} = x^2z^3\,\vec{i} + 2xyz^3\,\vec{j} + xz^4\,\vec{k}$$

and S is the surface of the box with vertices $(\pm1, \pm2, \pm3)$.

b) (**15 points**)

Evaluate

$$\iint_S \vec{F} \bullet d\,\vec{S} = \iint_S \vec{F} \bullet \vec{n} dS$$

where

$$\vec{F} = x^2\vec{i} + xy\vec{j} + z\vec{k}$$

and S is the portion of the paraboloid $z = x^2 + y^2$ below $z = 1$, oriented with the normal pointing downward.

2011 Problem 5

a) (**13 points**)

Use Stokes' Theorem to evaluate

$$\iint_S \operatorname{curl} \vec{F} \cdot d\vec{S}$$

where $\vec{F} = z^2\vec{i} - 3xy\vec{j} + x^3y^3\vec{k}$ and S is the part of $z = 5 - x^2 - y^2$ above the plane $z = 1$. Assume S is oriented upwards.

b) (**12 points**)

Use the divergence theorem to evaluate

$$\iint_S \vec{F} \cdot d\vec{S}$$

where $\vec{F} = xy\vec{i} - \frac{1}{2}y^2\vec{j} + z\vec{k}$ and the surface S consists of the three surfaces, $z = 4 - 3x^2 - 3y^2,\ 1 \le z \le 4,$ on the top, $x^2 + y^2 = 1,\ 0 \le z \le 1$ on the sides, and $z = 0$ on the bottom.

2010 problem 5

(**25 points**)

Verify the Divergence Theorem for the vector field $\vec{F} = y\vec{i} + yz\vec{j} + z^2\vec{k}$ where S is the surface of the solid E bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 5$. a) (15 points)

2009 problem 5

Verify Stokes' Theorem for the vector field $\vec{F} = -y\vec{i} + 2x\vec{j} + (z + x)\vec{k}$ over the upper hemisphere $x^2 + y^2 + z^2 = 1,\ z \ge 1,$ oriented by the outward pointing normal $\vec{n}$.