Selected problems

2009, 2010 & 2011 Ma 227 Final Exams

Multiple Integrals

2011 problem 2b

Consider the triple integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{1+\sqrt{1-x^2-y^2}} (x^2+y^2+z^2)\,dzdydx.$$

i. Describe and sketch the region of integration.

ii. Give an equivalent triple integral in cylindrical coordinates.

iii. Give an equivalent triple integral in spherical coordinates.

2010 Problem 3

Consider the triple integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (x^2+y^2+z^2)\,dzdydx.$$

i. Describe and sketch the region of integration.

ii. Give an equivalent triple integral in rectangular coordinates in a different order of integration.

iii. Give an equivalent triple integral in cylindrical coordinates.

iv. Give an equivalent triple integral in spherical coordinates.

v. Use any of your equivalent triple integrals to evaluate the integral.

2009 Problem 3

a) (**12 points**)

Let R be the region in the first octant bounded by the plane $x + 2y + 3z = 6$. Sketch the region R. Set up three iterated integrals for the volume with the orders of integration as specified below.

$$\iiint dzdydx \quad , \quad \iiint dydxdz \quad , \quad \iiint dxdzdy$$

b) (**13 points**)

The integral below represents the volume of a solid. Describe the solid. Evaluate the integral.

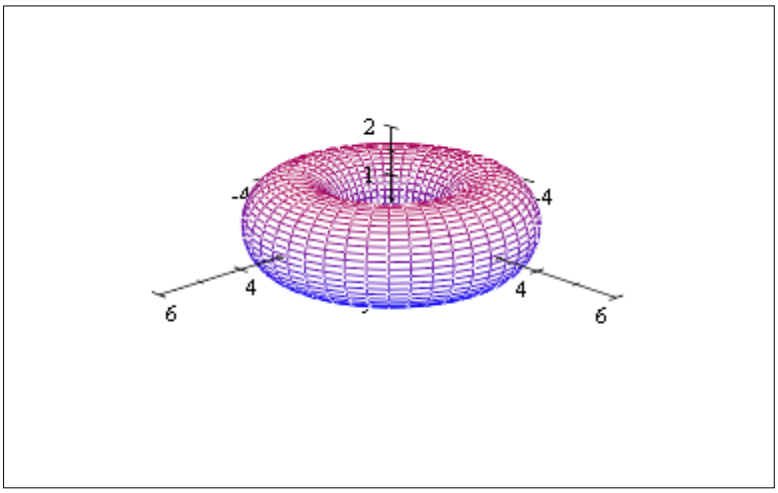
$$\int_{-2}^{2} \int_0^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} dzdydx$$

2011 Problem 4b

The figure below shows the torus obtained by rotating about the z-axis the circle in the xz-plane with center $(2,0,0)$ and radius 1.
Parametric equations for the torus are

$$x = 2\cos\theta + \cos\alpha\cos\theta$$
$$y = 2\sin\theta + \cos\alpha\sin\theta$$
$$z = \sin\alpha$$
$$0 \le \alpha \le 2\pi, \quad 0 \le \theta \le 2\pi.$$

θ is the usual polar angle around the z axis and α is the angle around the circle in the xz-plane. Find the surface area of the torus.



2011 Problem 7

a) (**13 points**)
The integral

$$\int_{-2}^{1}\int_{y^2}^{2-y} dxdy$$

gives the area of a region R in the x,y –plane. Sketch R and then give another expression for the area of R with the order of integration reversed. Do *not* evaluate this expression.

b) (**12 points**)
Find the volume of the region that lies under the sphere $x^2 + y^2 + z^2 = 9$, above the plane $z = 0$ and inside the cylinder $x^2 + y^2 = 5$. Sketch the solid.

2010 Problem 7

a) (**13 points**)

The integral

$$\int_{-2}^{1}\int_{y^2}^{2-y} dxdy$$

gives the area of a region R in the x,y –plane. Sketch R and then give another expression for the area of R with the order of integration reversed. Do *not* evaluate this expression.

b) (**12 points**)

Find the volume of the region that lies under the sphere $x^2+y^2+z^2=9,$ above the plane $z=0$ and inside the cylinder $x^2+y^2=5.$ Sketch the solid.

2009 Problem 7

a) (**13 points**)

Calculate

$$\iint_R (x^2+y^2)^{-2} dA$$

where R is the part of the circle centered at $(1,0)$ of radius 1 to the right of the line $x=1$ in the first quadrant. Be sure to sketch R.

b) (**12 points**)

Evaluate

$$\iiint_E z dV$$

where E is the region within the cylinder $x^2+y^2=4,$ where $0\le z\le y$.