Homework Ma227 Fall 2013
Due October 4, 2013

13.2 Pg 921-922
Problems: 3, 7, 11, 17, 19, 21
#3
$\int_c xy^4 ds \qquad C$ is the right half of the circle $x^2 + y^2 = 16$

In this case: $\frac{-\pi}{2} \le t \le \frac{\pi}{2}$
$x(t) = 4\cos t \quad ; \frac{dx}{dt} = -4\sin t$
$y(t) = 4\sin t \quad ; \frac{dy}{dt} = 4\cos t$

$$\int xy^4 ds = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (4\cos t)(4\sin t)^4 \sqrt{16\sin^2 t + 16\cos^2 t}\,)dt = 4(\tfrac{4^5}{5})\sin^5 t \;|_{\frac{-\pi}{2}}^{\frac{\pi}{2}} = \frac{2(4)^6}{5} = 1638.4$$

7)
$\int_c xydx + (x-y)dy \qquad C$ consists of line segments from $(0,0)$ to $(2,0)$ and $(2,0)$ to $(3,0)$

For $0 \le t \le 2$: $\quad x(t) = t \quad dx = dt$
$\qquad y(t) = 0 \quad dy = 0$
For $2 \le t \le 3 \quad x(t) = t \quad dx = dt$
$\qquad y(t) = 2t \quad dy = 2dt$

$$\int xydx + (x-y)dy = \int_0^2 (2)(0)dt + (t)(0)dt + \int_2^3 (2t-4)tdt + (t-(2t-4)2dt$$
$$= \int_2^3 (2t^2 - 6t + 8)dt = \frac{2}{3}t^3 - 3t^2 + 8t \;|_2^3 = \frac{17}{3}$$

11)
$\int_c xe^{yz}ds,\ C$ is the line segment from $(0,0,0)$ to $(1,2,3)$
$x(t) = t$
$y(t) = 2t$
$z(t) = 3t$
$0 \le t \le 1$

$$\int_c = xe^{yz}ds = \int_0^1 te^{(2t)(3t)}\sqrt{1^2 + 2^2 + 3^2}\,dt$$
$$= \sqrt{14}\int_0^1 te^{6t^2}dt = \sqrt{14}\left[\frac{1}{12}e^{6t^2}\right]_0^1 = \frac{\sqrt{14}}{12}(e^6 - 1)$$

17) (a) Along the line $x = -3$ the vectors of $\vec{F}$ have positive y –components. so since the path goes upward, the integrand $\vec{F} \cdot \vec{T}$ is always positive. Therefore $\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_1} \vec{F} \cdot \vec{T} ds$ is positive.

(b) All of the (non-zero) field vectors along the circle with radius 3 are pointed in the clockwise direction, that is, opposite the direction to the path. So $\vec{F} \cdot \vec{T}$ is negative, and therefore $\int_{C_2} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot \vec{T} ds$ is negative.

19)

$r(t) = 11t^4\vec{i} + t^3 j$ and $F(x,y = xy\vec{i} + 3y^2\vec{j}$ so $F(r(t)) = (11t^4)(t^3)\vec{i} + 3(t^6)\vec{j} = 11t^7\vec{i} + 3t^6\vec{j}$

$r'(t) = 44t^3\vec{i} + 3t^2\vec{j}$

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^1 \langle F(r(t) \cdot r'(t) dt = \int_0^1 (11t^7 \cdot 44t^3 + 3t^6 \cdot 3t^2) dt$$

$$= \int_0^1 (484t^{10} + 9t^8) dt = [44t^{11} + t^9]_0^1 = 45$$

21)

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^1 \langle \sin t^3, \cos(-t^2), t^4 \rangle \cdot \langle 3t^2, -2t, 1 \rangle dt$$

$$= \int_0^1 (3t^2 \sin t^3 - 2t \cos t^2 + t^4) dt$$

$$= \left[-\cos t^3 - \sin t^2 + \frac{1}{5} t^5 \right]_0^1 = \frac{6}{5} - \cos 1 - \sin 1$$

Pg 932-933 13.3

Problems: 5, 7, 15,19,23

5) $\vec{F}(x,y) = \underset{P}{e^x \cos y}\, \vec{i} + \underset{Q}{e^x \sin y}\, \vec{j}$

$\frac{\partial P}{\partial y} = -e^x \sin y$

$\frac{\partial P}{\partial x} = e^x \sin y$

Since $\frac{\partial Q}{\partial x} \neq \frac{\partial Q}{\partial y}$ $\quad \vec{F}$ is **NOT** conservative

7) $\vec{F}(x,y) = \underset{P}{(ye^x + \sin y)}\, \vec{i} + \underset{Q}{(e^x + x \cos y)}\, \vec{j}$

$\frac{\partial P}{\partial y} = e^x + \cos y$

$\frac{\partial P}{\partial x} = e^x + \cos y$

F is conservative

$f_x(x,y) = ye^x + \sin y \Rightarrow f(x,y) = ye^x + x \sin y + g(y)$

$f_y(x,y) = e^x + x \cos y + g'(y)$

$f_x(x,y) = xy \cos xy + \sin xy$ so $g(x) = C$

$\Rightarrow f(x,y) = ye^x + x \sin y + C$

15)

$\vec{F}(x,y,z) = yz\,\vec{i} + xz\,\vec{j} + (xy + 2z)\vec{k}$

a)

$f_x(x,y,z) = yz \Rightarrow f(x,y,z) = yz + g(y,z)$
$f_y(x,y,z) = xz + g_y(y,z) = xz \Rightarrow g_y(y,z) = 0$
$g(y,z) = h(z)$

$$f(x,y,z) = xyz + h(z); f_z(x,y,z) = xyz + h'(z)$$
$$= xy + 2z \Rightarrow h'(z) = 2z \Rightarrow h(z) = z^2 + K$$

$\Rightarrow$

$$f(x,y,z) = xyz + z^2 + K$$

b)
$\int_C \vec{F} \cdot d\vec{r} = f(4,6,3) - f(1,0,-2) = 81 - 4 = 77$

19) $\int \tan y dx + x \sec^2 y dy$
Let $\vec{F}(x,y) = (\tan y)\vec{i} + (\sec^2 y)\vec{j}$
Then $f(x,y) = x \tan y$ is a potential function for F so it is conservative and thus its line integral is independent of its path. Hence:
$\int_C \tan y dx + x \sec^2 y dy = \int_C F \cdot dr = f(2, \frac{\pi}{4}) - f(-1,0) = 2 \tan \frac{\pi}{4} - \tan 0 = 2$

23) $\vec{F}(x,y) = (2y^{\frac{3}{2}})\vec{i} + (3x\sqrt{y})\vec{j} \qquad P(1,1), Q(2,4)$
$W = \int \vec{F} \cdot d\vec{r}$
$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} : 3\sqrt{y} = 3\sqrt{y}$
$f : \vec{F} = \nabla f$
$f(x,y) = 2xy^{\frac{3}{2}} + g(y) \Rightarrow f_y(x,y) = 3xy^{\frac{1}{2}} + g'(y).$
$f_y(x,y) = 3x\sqrt{y}$ so $g'(y) = 0 = g(y)$
$f(x,y) = 2xy^{\frac{3}{2}}$
$W = \int \vec{F} \cdot d\vec{r} = f(2,4) - f(1,1) = 2(2)(8) - 2(1) = 30$