Ma 227 Homework 5 Solutions Fall 2013
Due 8/30/2013

12.2 Pg. 843 Problems 5,7,25, 27

5)
$$\int_0^2 \int_0^{\frac{\pi}{2}} x\sin y\,dy\,dx = \int_0^2 x\,dx \int_0^{\frac{\pi}{2}} \sin y\,dy = \left[\frac{x^2}{2}\right]_0^2 [-\cos y]_0^{\frac{\pi}{2}} = (2-0)(0+1) = 2$$

7)
$$\int_0^2 \int_0^1 (2x+y)^8 dx\,dy = \int_0^2 \left[\frac{1}{2}\frac{(2x+y)^9}{9}\right]_{x=0}^{x=1} dy$$

Use substitituon: $u = 2x + y \Rightarrow dx = \frac{1}{2}du$

$$\frac{1}{18}\int_0^2 \left[(2+y)^9 - (0+y)^9\right]dy = \frac{1}{18}\left[\frac{(2+y)^{10}}{10} - \frac{y^{10}}{10}\right]_0^2 = \frac{1}{18}[(4^{10} - 2^{10}) - (2^{10} - 0^{10})]$$
$$= \frac{1{,}046{,}528}{180} = \frac{261{,}632}{45}$$

25) Find the volume of the solid that lies under the plane $3x + 2y + z = 12$ and above the rectangle $R = \{(x,y)|0 \le x \le 1, -2 \le y \le 3\}$

$z = 12 - 3x - 2y$

$$V = \iint_R (12 - 3x - 2y)dA = \int_{-2}^3 \int_0^1 (12 - 3x - 2y)dx\,dy =$$
$$\int_{-2}^3 [12x - \tfrac{3}{2}x^2 - 2xy]_{x=0}^{x=1} dy = \int_{-2}^3 (\tfrac{21}{2} - 2y)dy$$

$$= [\tfrac{21}{2}y - 2y^2]_{-2}^3 = \frac{95}{2}$$

27) Find the volume of the solid that lies under the elliptic paraboloid $\frac{x^2}{4} + \frac{y^2}{9} + z = 1$ and above the rectangle $R = [-1,1] \times [-2,2]$

$z = 1 - \frac{x^2}{4} - \frac{y^2}{9}$

$$V = \iint_R \left(1 - \frac{x^2}{4} - \frac{y^2}{9}\right)dV = \int_{-2}^2 \int_{-1}^1 \left(1 - \frac{x^2}{4} - \frac{y^2}{9}\right)dx\,dy = 4\int_0^2 \int_0^1 \left(1 - \frac{x^2}{4} - \frac{y^2}{9}\right)dx\,dy$$
$$= 4\int_0^2 \left[x - \frac{1}{12}x^3 - \frac{1}{9}y^2x\right]_{x=0}^{x=1} dy = 4\int_0^2 (\tfrac{11}{12} - \tfrac{1}{9}y^2)dy = 4\left[\frac{11}{12}y - \frac{1}{27}y^3\right]_0^2 = 4 \bullet \frac{83}{54} = \frac{166}{27}$$

12.3 Pg. 850-51 Problems #5, 7, 17, 21, 25, 41, 45

5)
$$\int_0^{\frac{\pi}{2}} \int_0^{\cos\theta} e^{\sin\theta} dr\,d\theta = \int_0^{\frac{\pi}{2}} re^{\sin\theta}]_0^{\cos\theta} d\theta = \int_0^{\frac{\pi}{2}} \cos\theta e^{\sin\theta} d\theta = e^{\sin\theta}]_0^{\frac{\pi}{2}} = e - 1$$

7)

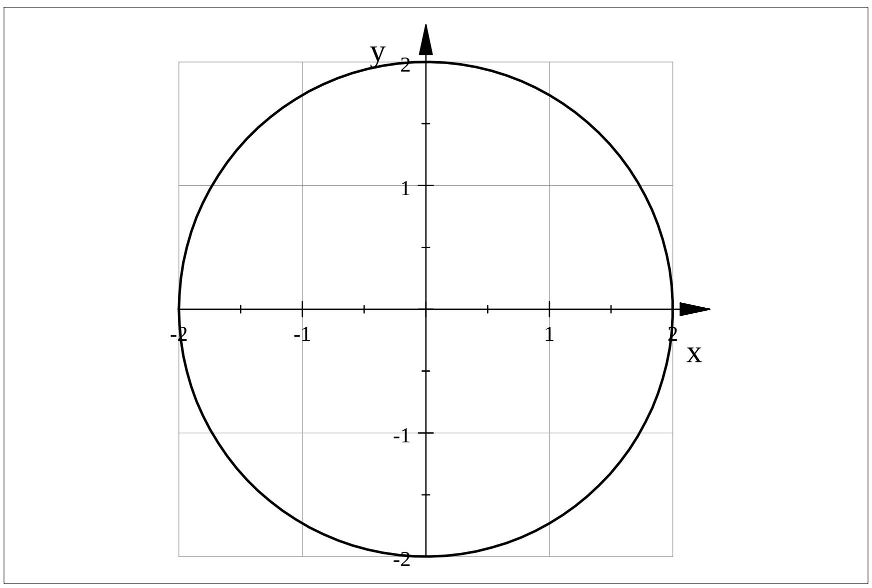
$\int\int_D y^2 dA = \int_{-1}^{1}\int_{-y-2}^{y} y^2 dxdy = \int_{-1}^{1}[xy^2]_{x=-y-2}^{x=y}dy = \int_{-1}^{1} y^2[y-(-y-2)]dy = \int_{-1}^{1}(2y^3+2y^2)dy =$

17)

$\int\int_D x\cos y\, dA \qquad D: y=0, y=x^2, x=1$

$= \int_0^1\int_0^{x^2}(x\cos y)$
$dxdy = \int_0^1 (x\sin y)]_0^{x^2} dx = \int_0^1 (x\sin x^2)dx = -\frac{1}{2}\cos x^2]_0^1 = -\frac{1}{2}\cos(1)+\frac{1}{2} = \frac{1}{2}(1-\cos(1))$

21)
$\int\int_D (2x-y)dA \qquad D: \; x^2+y^2=4$
The graph of D is shown below. Graph 2D Polare was used to get it.
$r=2$

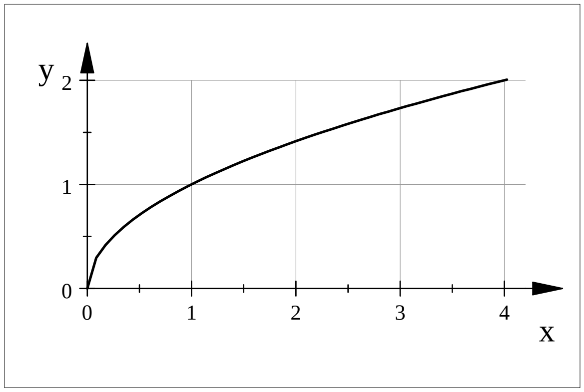


$= \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}(2x-y)dydx = \int_{-2}^{2}(2xy-\frac{1}{2}y^2)]_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\; dx$
$= \int_{-2}^{2} 2x\sqrt{4-x^2} - \frac{1}{2}(4-x^2) - 2x(-\sqrt{4-x^2}) + \frac{1}{2}(4-x^2)$
$dx = \int_{-2}^{2}(4x\sqrt{4-x^2})dx = \frac{4}{3}(4-x^2)^{\frac{3}{2}}]_{-2}^{2} = 0$

25)
$V = \int_1^2\int_1^{7-3y} xydxdy = \int_1^2[\frac{1}{2}x^2y]_{x=1}^{x=7-3y}dy = \frac{1}{2}\int_1^2(48y-42y^2+9y^3)dy = \frac{1}{2}[24y^2-14y^3+\frac{9}{4}y^4$

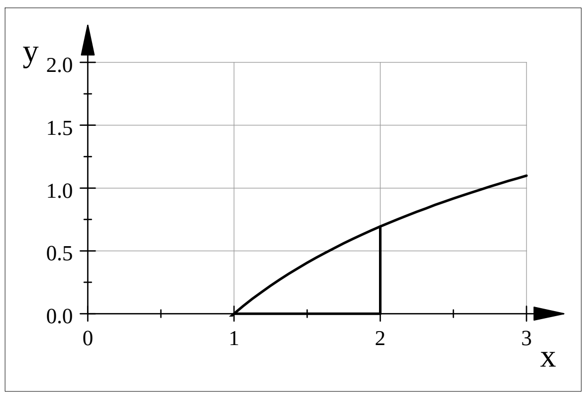
41)

$\sqrt{x}$



Because the region of integration is
$D = \{(x,y)|0 \leq y \leq \sqrt{x}, 0 \leq x \leq 4\} = \{(x,y)|y^2 \leq x \leq 4, 0 \leq y \leq 2\}$
we have
$\int_0^4 \int_0^{\sqrt{x}} f(x,y)dydx = \int\int_D f(x,y)dA = \int_0^2 \int_{y^2}^4 f(x,y)dxdy$

45)



Because the region of integration is
$D = \{(x,y)|0 \leq y \leq \ln x, 1 \leq x \leq 2\} = \{(x,y)|e^y \leq x \leq 2, 0 \leq y \leq \ln 2\}$
we have
$\int_1^2 \int_0^{\ln x} f(x,y)dydx = \int_0^{\ln 2} \int_{e^y}^2 f(x,y)dxdy$