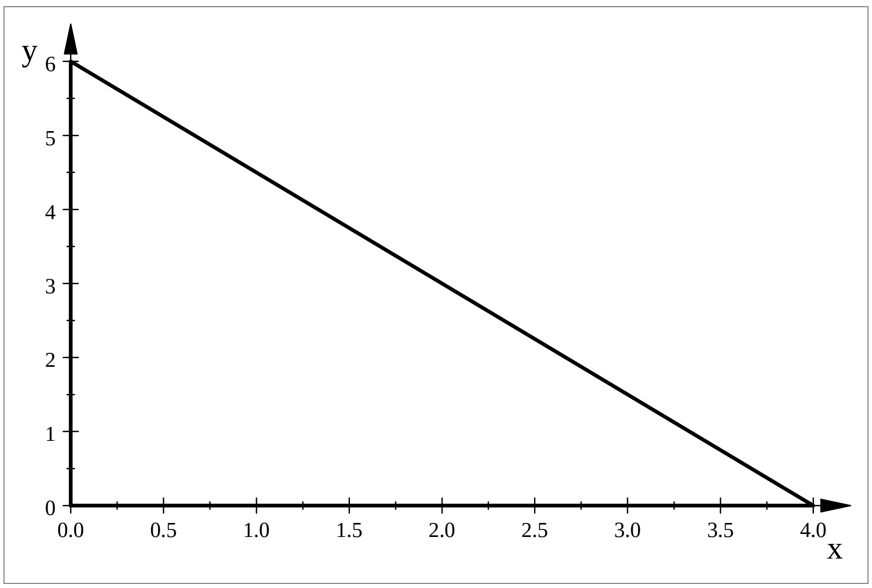
# Ma 227 Fall 2013 Due 9/20/2013

Section 12.6 #1, 2, 3, 5

1) Find the surface area for:
The part of the plane $x+2y+3z=1$ that lies inside the cylinder $x^2+y^2=3$.
Here $z=f(x,y)=\frac{1}{3}-\frac{1}{3}x-\frac{2}{3}y$, so by formula 6 the area of the surface is

$$A(S)=\iint_D\sqrt{1+(\frac{\partial z}{\partial x})^2+(\frac{\partial z}{\partial y})^2}\,dA=\iint_D\sqrt{1+\left(-\frac{1}{3}\right)^2+\left(-\frac{2}{3}\right)^2}\,dA=\frac{\sqrt{14}}{3}\iint_D dA=\frac{\sqrt{14}}{3}A($$

2)Find the surface area for:
The part of the plane $2x-5y+z=10$ that lies above the triangle with vertices $(0,0),(0,6),(4,0)$. The triangle is drwan below.
$(0,0,0,6,4,0)$




$z=f(x,y)=10-2x+5y$ and $D$ is the trianglur region $\{(x,y)|0\le x\le 4, 0\le y\le 6-\frac{3}{2}x\}$ , so by formula 6

$$A(S)=\iint_D\sqrt{1+(\frac{\partial z}{\partial x})^2+(\frac{\partial z}{\partial y})^2}\,dA=\iint_D\sqrt{1+(-2)^2+(5)^2}\,dA=\sqrt{30}\iint_D dA=\sqrt{30}\,A(D)=$$

3)Find the surface area for:
The part of the plane $3x+2y+z=6$ that lies in the first octant.
$z=f(x,y)=6-3x-2y$ which intersects the $xy$ –plane in the line $3x+2y=6$ , so $D$ is the triangular region given by $\{(x,y)|0\le x\le 2, 0\le y\le 3-\frac{3}{2}x\}$. Thus:
$A(S)=\iint_D\sqrt{1+(-3)^2+(-2)^2}\,dA=\sqrt{14}\iint_D dA=\sqrt{14}\,A(D)=\sqrt{14}\,(\frac{1}{2})(2)(3)=3\sqrt{14}$

5) The part of the cone $z = \sqrt{x^2+y^2}$ that lies between the plane $y = x$ and the cylinder $y = x^2$

$z = f(x,y) = \sqrt{x^2+y^2}$. Then

$$z_x = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}(2x) = \frac{x}{\sqrt{x^2+y^2}}$$

$$z_y = \frac{y}{\sqrt{x^2+y^2}}$$

and

$$\sqrt{1+(\frac{\partial z}{\partial x})^2+(\frac{\partial z}{\partial y})^2} = \sqrt{1+\frac{x^2}{x^2+y^2}+\frac{y^2}{x^2+y^2}} = \sqrt{1+\frac{x^2+y^2}{x^2+y^2}} = \sqrt{2}$$

Here $D$ is given by $\{(x,y)|0 \le x \le 1, x^2 \le y \le x\}$, so by formula 6

$$A(S) = \int\int_D \sqrt{2}\,dA = \sqrt{2}\int_0^1\int_{x^2}^x dydx = \sqrt{2}\int_0^1(x-x^2)dx = \sqrt{2}\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1 = \sqrt{2}\left(\frac{1}{2}-\frac{1}{3}\right) = \frac{\sqrt{2}}{6}$$

Section 13.1 - p. 912 #21, 23, 25

21.

$$f(x,y) = xe^{xy}$$

$$\begin{aligned}\nabla f(x,y) &= f_x(x,y)\vec{i}+f_y(x,y)\vec{j}\\ &= (xe^{xy}y+e^{xy})\vec{i}+(xe^{xy}x)\vec{j} = (xy+1)e^{xy}\vec{i}+x^2e^{xy}\vec{j}\end{aligned}$$

23. Find the gradient vector field of $f$

$f(x,y,z) = \sqrt{x^2+y^2+z^2}$

gradient vector field $= f_x(x,y,z)\vec{i}+f_y(x,y,z)\vec{j}+f_z(x,y,z)\vec{k}$

$= \frac{\partial}{\partial x}\sqrt{x^2+y^2+z^2}\,\vec{i}+\frac{\partial}{\partial y}\sqrt{x^2+y^2+z^2}\,j+\frac{\partial}{\partial z}\sqrt{x^2+y^2+z^2}\,\vec{k} =$

$= \frac{x}{\sqrt{x^2+y^2+z^2}}i\_+\frac{y}{\sqrt{x^2+y^2+z^2}}\vec{j}+\frac{z}{\sqrt{x^2+y^2+z^2}}\vec{k}$

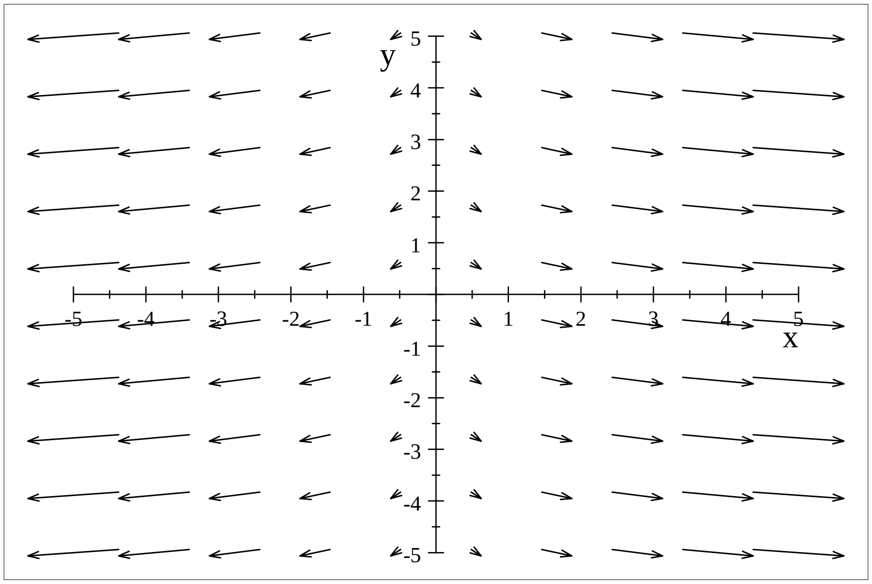
SNB check:

$\nabla\left(\sqrt{x^2+y^2+z^2}\right) = \left(\frac{1}{\sqrt{x^2+y^2+z^2}}x, \frac{1}{\sqrt{x^2+y^2+z^2}}y, \frac{1}{\sqrt{x^2+y^2+z^2}}z\right)$

25. Find the gradient vector field $\nabla f$ of $f$ and sketch it.

$f(x,y) = x^2-y$. $\nabla f = 2x\vec{i}-\vec{j}$. The length of $\nabla f$ is $\sqrt{4x^2+1}$. When $x \ne 0$ the vectors point away from the $y-axis$ in a slightly downward direction with length that increases as the distance distance from the $y-axis$ increases.

Using SNB we have for the gradient of $x^2-y$,




Section 13.5 - p. 947 #1, 5, 15, 17, 21

Find (a) the curl and (b) the divergence of the vector field.

1. $F(x,y,z) = xyz\vec{i} - x^2y\vec{k}$

a. calculation of curl

$$\operatorname{curl} F = \nabla \times F = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xyz & 0 & -x^2y \end{vmatrix} = \vec{i}(-\tfrac{\partial}{\partial y}x^2y - 0) - \vec{j}(-\tfrac{\partial}{\partial x}x^2y - \tfrac{\partial}{\partial z}xyz) + \vec{k}(0 - \tfrac{\partial}{\partial y}xyz)$$

$$= (-x^2)\vec{i} - \vec{j}(-2xy - xy) + \vec{k}(0 - xz)$$

$$= -x^2\vec{i} + 3xy\vec{j} - xz\vec{k}$$

SNB check: $\nabla \times (e^x \sin y, e^x \cos y, z) = (0,0,0)$

b. divergence :

$$\operatorname{div} F = \nabla \bullet F = \tfrac{\partial}{\partial x}(xyz) + \tfrac{\partial}{\partial y}(0) + \tfrac{\partial}{\partial z}(-x^2y) = yz + 0 + 0 = yz$$

5. a. calculation of curl

$$\operatorname{curl} F = \nabla \times F = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{\sqrt{x^2+y^2+z^2}} & \frac{y}{\sqrt{x^2+y^2+z^2}} & \frac{z}{\sqrt{x^2+y^2+z^2}} \end{vmatrix} = \frac{1}{(x^2+y^2+z^2)^{\frac{3}{2}}}\Big[(-yz + yz)\vec{i} + (-xz + xz)$$

b. divergence :

$$\operatorname{div} F = \nabla \cdot F = \frac{\partial}{\partial x}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right) + \frac{\partial}{\partial y}\left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right) + \frac{\partial}{\partial z}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

$$= \frac{x^2+y^2+z^2-x^2}{(x^2+y^2+z^2)^{\frac{3}{2}}} + \frac{x^2+y^2+z^2-y^2}{(x^2+y^2+z^2)^{\frac{3}{2}}} + \frac{x^2+y^2+z^2-z^2}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

$$= \frac{2x^2+2y^2+2z^2}{(x^2+y^2+z^2)^{\frac{3}{2}}} = \frac{2}{\sqrt{x^2+y^2+z^2}}$$

15. Determine whether or not the vector field is conservative. If it is conservatice find the function $f$ such that $F = \nabla f$

$\vec{F}(x,y,z) = 2xyi + (x^2 + 2yz)j + y^2k$

to check if the vector field is conservative, calculate the curl for the field.

$$\operatorname{curl}\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & x^2+2yz & y^2 \end{vmatrix} = (2y-2y)i - (0-0)j + (2x-2x)k = 0$$

and $\vec{F}$ is defined on all of $R^3$ with component functions which have continuous partial derivatives, so by Theorem 4, $\vec{F}$ is conservative. Thus there exists $f(x,y,z)$ such that $\nabla f = \vec{F}$. now

$$f_x = 2xy$$

so

$$f = x^2y + g(y,z)$$

Hence

$$f_y = x^2 + g_y(y,z) = x^2 + 2yz$$

$\Rightarrow$

$$g_y(y,z) = y^2z + h(z)$$

$\Rightarrow$

$$f = x^2y + y^2z + h(z)$$

Then

$$f_z = y^2 + h'(z) = y^2$$

$h(z) = k,$ where $k$ is a constant. Thus

$$f(x,y,z) = x^2y + y^2z + k$$

17. Determine whether or not the vector field is conservative. If it is conservatice find the function $f$ such that $F = \nabla f$

$\vec{F}(x,y,z) = ye^{-x}\vec{i} + e^{-x}\vec{j} + 2z\vec{k}$

calculation of curl:

$$\text{curl}\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ye^{-x} & e^{-x} & 2z \end{vmatrix} = \vec{i}(\frac{\partial}{\partial y}2z - \frac{\partial}{\partial z}e^{-x}) - \vec{j}(\frac{\partial}{\partial x}2z - \frac{\partial}{\partial z}ye^{-x}) + \vec{k}(\frac{\partial}{\partial x}e^{-x} - \frac{\partial}{\partial y}ye^{-x})$$

$$= -2ke^{-x} \neq 0$$

which is not equal to 0, so its not conservative.

21.

Calculate the curl of $\vec{F} = f(x)\vec{i} + g(y)\vec{j} + h(z)\vec{k}$. If this is zero, then the vector field is irrotational.

$$\nabla \times (f(x), g(y), h(z)) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f(x) & f(y) & f(z) \end{vmatrix} = (0, 0, 0)$$

Therefore it is irrotational.