Ma 227 **Exam I A Solutions** **10/4/12**

Name: ________________________________

Lecture Section: ______

I pledge my honor that I have abided by the Stevens Honor System. ___________________________________

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.
There is a table of integrals on the last page of the exam.

Score on Problem #1 ________

#2a ________

#2b ________

#3 ________

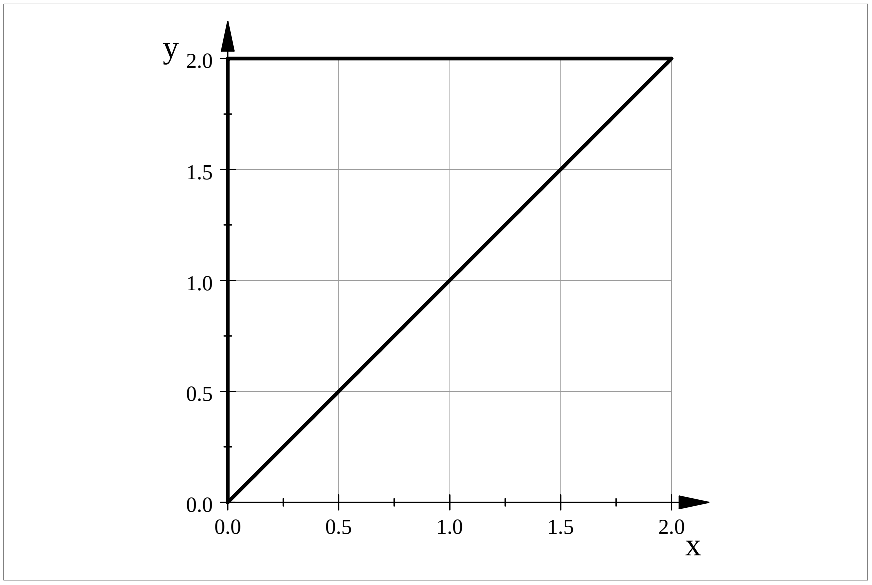
#4 ________

Total Score ________

1 [25 **pts**.] Evaluate

$$\int_0^2 \int_x^2 e^{y^2} dydx$$

Sketch the region of integration.
Solution: The region of integration is shown below.



To evaluate the integral we reverse the order of integration.

$$\int_0^2 \int_x^2 e^{y^2} dydx = \int_0^2 \int_0^y e^{y^2} dxdy$$

$$= \int_0^2 ye^{y^2} dy = \left. \frac{e^{y^2}}{2} \right|_0^2 = \frac{1}{2}\left[e^4 - 1\right]$$

2 a [20 **pts**.] Evaluate

$$\iint_R \left(x^2 + y^2\right) dA$$

where R is the circle $x^2 + y^2 = 2x$. Sketch R.
Solution: The equation of the circle may be put in standard form.

$$x^2 + y^2 - 2x = (x-1)^2 + y^2 - 1 = 0$$

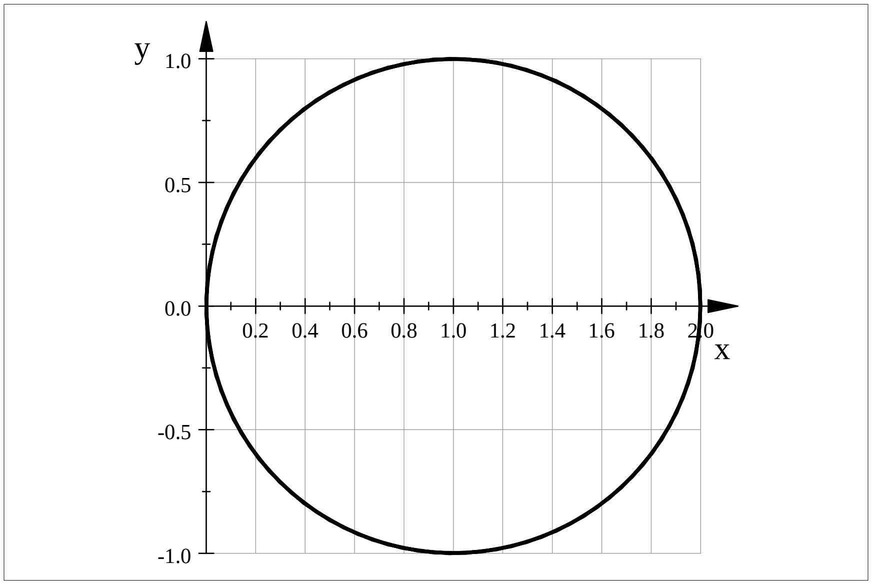
or

$$(x-1)^2 + y^2 = 1$$

This circle is centered at $(1,0)$ and has radius 1. In polar coordinates since $x^2 + y^2 = r^2$ and $x = r\cos\theta$, we have the eqaution

$$r = 2\cos\theta$$

$2\cos\theta$



Since the circle is only in the first and fourth quadrants, then

$$\iint_R \left(x^2+y^2\right)dA = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{2\cos\theta} r^2 r\,dr\,d\theta$$
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left.\frac{r^4}{4}\right|_0^{2\cos\theta} d\theta$$
$$= 4\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4\theta\, d\theta$$
$$= 4\left[\frac{3}{8}\theta + \frac{1}{4}\sin 2\theta + \frac{1}{32}\sin 4\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{3}{2}\pi$$

2 b [15 **pts**.] Give an integral in polar coordinates for the surface area of the part of the hyperboloid $z = xy$ that lies within the cylinder $x^2+y^2 = 1$. DO NOT EVALUATE THIS INTEGRAL.

Solution: Here $f(x,y) = xy$ and $0 \le x^2+y^2 \le 1$. Thus $f_x = y, f_y = x$ and therefore

$$A(S) = \iint_D \sqrt{1+f_x^2+f_y^2}\, dA$$
$$= \iint_D \sqrt{1+x^2+y^2}\, dA$$
$$= \int_0^{2\pi}\int_0^1 \sqrt{1+r^2}\, r\,dr\,d\theta$$

3 [20 **pts**.] Use cylindrical coordinates to set up an iterated triple integral for the volume of the solid that lies within both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$. DO NOT EVALUATE THIS INTEGRAL.

Solution: In cylindrical coordinates the volume E is the solid region within the cylinder $r = 1$ bounded above and below by the sphere $r^2 + z^2 = 4$. So

$$E = \left\{(r,\theta,z) \mid 0 \le \theta \le 2\pi, 0 \le r \le 1, -\sqrt{4-r^2} \le z \le \sqrt{4-r^2}\right\}$$

Hence the volume is given by

$$\iiint_E dV = \int_0^{2\pi}\int_0^1\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$$

4 [20 **pts**.] Use spherical coordinates to evaluate

$$\int_{-2}^{2}\int_0^{\sqrt{4-y^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2\sqrt{x^2+y^2+z^2}\,dz\,dx\,dy$$

Solution:

$$\int_{-2}^{2}\int_0^{\sqrt{4-y^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2\sqrt{x^2+y^2+z^2}\,dz\,dx\,dy$$

$$= \int_{=\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{\pi}\int_0^{2}(\rho\sin\phi\sin\theta)^2\sqrt{\rho^2}\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$= \int_{=\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^2\theta\,d\theta\int_0^{\pi}\sin^3\phi\,d\phi\int_0^{2}\rho^5\,d\rho$$

$$= \left[-\frac{1}{2}\cos\theta\sin\theta + \frac{1}{2}\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[-\frac{1}{3}\sin^2\theta\cos\theta - \frac{2}{3}\cos\theta\right]_0^{\pi}\left[\frac{\rho^6}{6}\right]_0^{2}$$

$$= \left[\frac{\pi}{2}\right]\left[\frac{2}{3} + \frac{2}{3}\right]\left[\frac{32}{3}\right] = \frac{64}{9}\pi$$

Table of Integrals

$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \sin^3 x dx = -\frac{1}{3}\sin^2 x \cos x - \frac{2}{3}\cos x + C$
$\int \cos^3 x dx = \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x + C$
$\int \sin^4 x dx = \frac{3}{8}x - \frac{3}{16}\pi - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$
$\int \cos^4 x dx = \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$
$\int te^{at} dt = \frac{1}{a^2}e^{at}(at - 1) + C$
$\int t^2 e^{at} dt = \frac{1}{a^3}e^{at}\left(a^2t^2 - 2at + 2\right) + C$