Ma 227 **Exam II Solutions** **11/6/06**

Name: ________________________________

Lecture Section: ______

I pledge my honor that I have abided by the Stevens Honor System. ___________________________________

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.
There is a table of integrals on the last page of the exam.

Score on Problem #1 ________

#2a ________

#2b ________

#3 ________

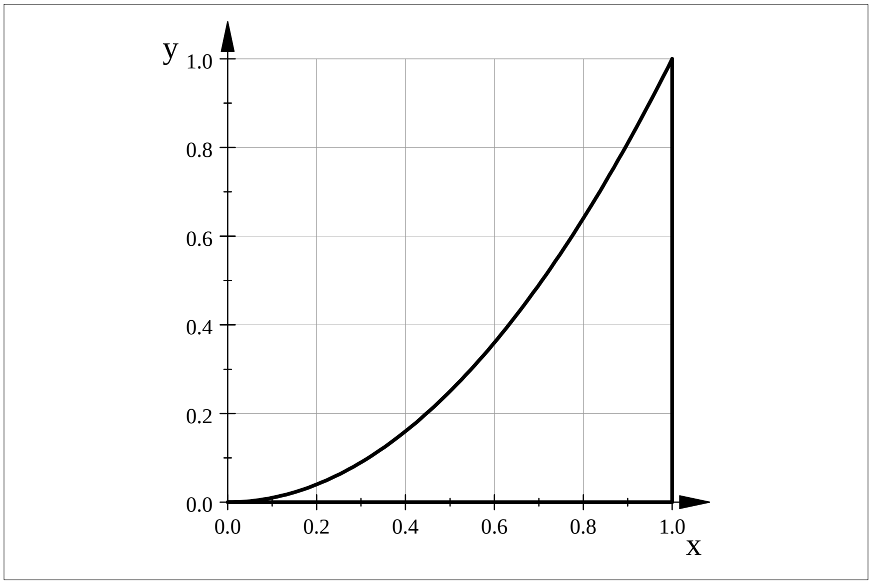
#4 ________

Total Score ________

1 [20 **pts**.] Evaluate the iterated integral

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3} dxdy$$

Solution: The region of integration is shown below.
x^2



We reverse the order of integration to be able to evaluate the intergral. Thus

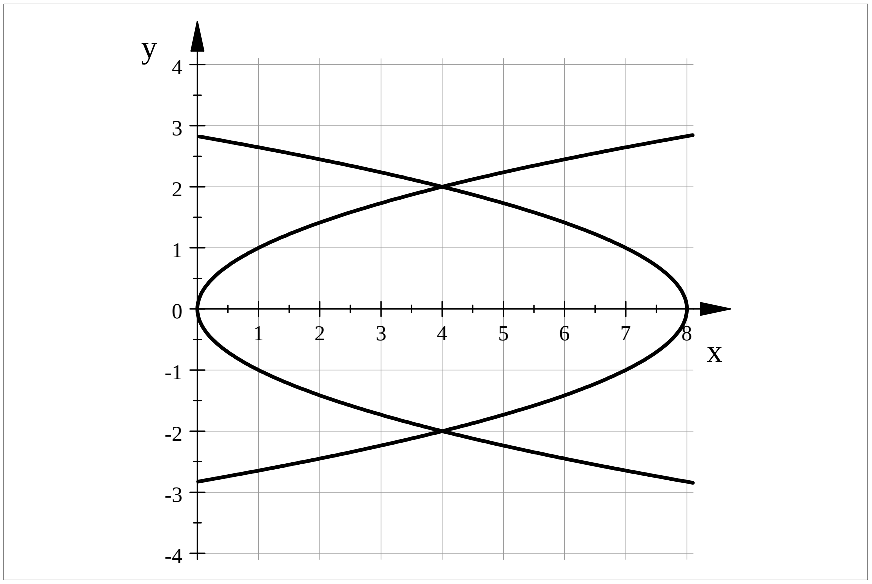
$$\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3} dxdy = \int_0^1 \int_0^{x^2} \frac{ye^{x^2}}{x^3} dydx$$

$$= \int_0^1 \frac{e^{x^2}}{x^3} \frac{y^2}{2} \Big]_0^{x^2} dx$$

$$= \frac{1}{2} \int_0^1 xe^{x^2} dx = \frac{1}{4} e^{x^2} \Big]_0^1 = \frac{1}{4}(e-1)$$

2 a [20 **pts**.] Give two integral expressions with different orders of integration for the area of the region R in the x,y –plane bounded by the parabolas $x = y^2$ and $x = 8 - y^2$. Sketch R. Do *not* evaluate these integral expressions.

Solution: The region of integration is shown below.
$x = y^2$



The two curves intersect when $y^2 = 8 - y^2$ or when $y^2 = 4,$ that is at $y = \pm 2, x = 4.$ Therefore

$$\text{Area} = \iint_R dA = \int_{-2}^{2}\int_{y^2}^{8-y^2} dxdy$$

$$= \int_0^4\int_{-\sqrt{x}}^{\sqrt{x}} dydx + \int_4^8\int_{-\sqrt{8-x}}^{\sqrt{8-x}} dydx$$

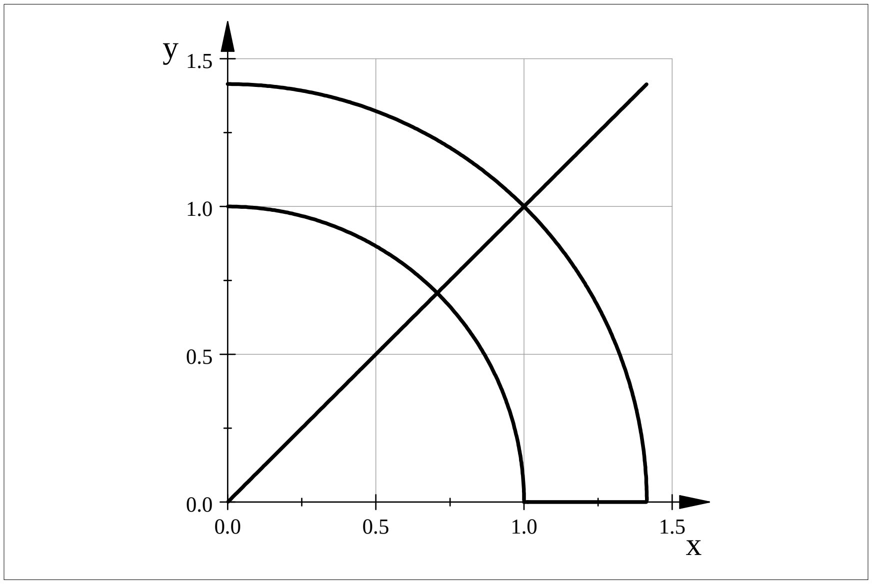
2 b $\left[20 \textbf{ pts.}\right]$ Evaluate

$$\iint_R xdA$$

where R is the region in the first quadrant that lies between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 2,$ and below the line $y = x.$ Sketch $R.$

Solution: The region of integration is shown below.

$x^2 + y^2 = 1$



We switch to polar coordinates since the area is bounded by two circles.

$$\iint_R x\,dA = \int_0^{\frac{\pi}{4}} \int_1^{\sqrt{2}} r\cos\theta\, r\,dr\,d\theta$$

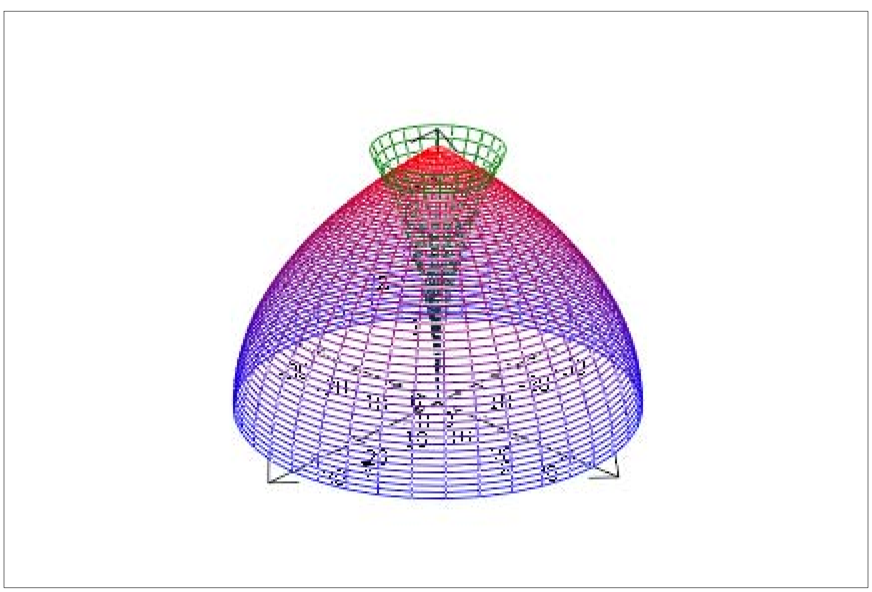
$$= \int_0^{\frac{\pi}{4}} \cos\theta \left.\frac{r^3}{3}\right]_1^{\sqrt{2}} d\theta = \frac{1}{3}\left[2^{\frac{3}{2}} - 1\right] \sin\theta]_0^{\frac{\pi}{4}}$$

$$= \frac{\sqrt{2}}{6}\left[2^{\frac{3}{2}} - 1\right] = \frac{2}{3} - \frac{1}{6}\sqrt{2}$$

3 $\left[20\right.$ **pts**.$\left.\right]$ Let V be the volume bounded by the paraboloids $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$. Give a triple integral expression in **cylindrical** coordinates for the volume of V. Sketch V. *Do not evaluate the integral you give.*

Solution:

$36 - 3r^2$



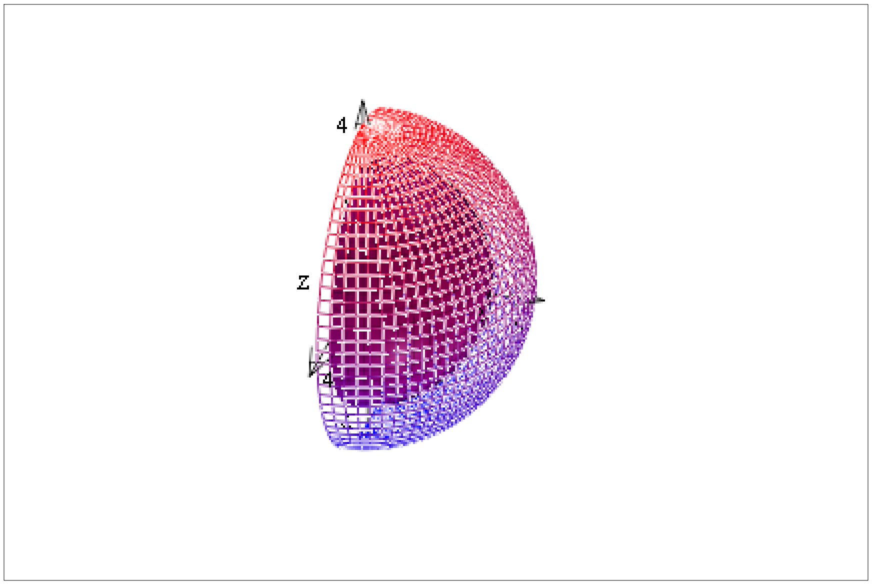
The paraboloids intersect when $x^2+y^2 = 36-3x^2-3y^2$, that is when $x^2+y^2 = 9$.
Thus in cylindrical coordinates we have

$$\text{Volume} = \iiint_V dV = \int_0^{2\pi}\int_0^3\int_{r^2}^{36-3r^2} r\,dz\,dr\,d\theta$$

4 [20 **pts**.] Give an integral expression in **spherical** coordinates for

$$\iiint_V x^2 dV$$

where V is bounded by the x,z –plane and the hemispheres $y = \sqrt{9-x^2-z^2}$ and $y = \sqrt{16-x^2-z^2}$.
Do not evaluate this expression.
Solution:
$\rho = 4$



$$\iiint_V x^2 dV = \int_0^{\pi}\int_0^{\pi}\int_3^4 (\rho\sin\phi\cos\theta)^2\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

Table of Integrals

$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \sin^3 x dx = -\frac{1}{3}\sin^2 x \cos x - \frac{2}{3}\cos x + C$
$\int \cos^3 x dx = \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x + C$