Ma 227 **Exam II Solutions** **11/8/10**

Name: ________________________________

Lecture Section: ______

I pledge my honor that I have abided by the Stevens Honor System. ___________________________________

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.
There is a table of integrals on the last page of the exam.

Score on Problem #1 ________

#2a ________

#2b ________

#3 ________

#4 ________

Total Score ________

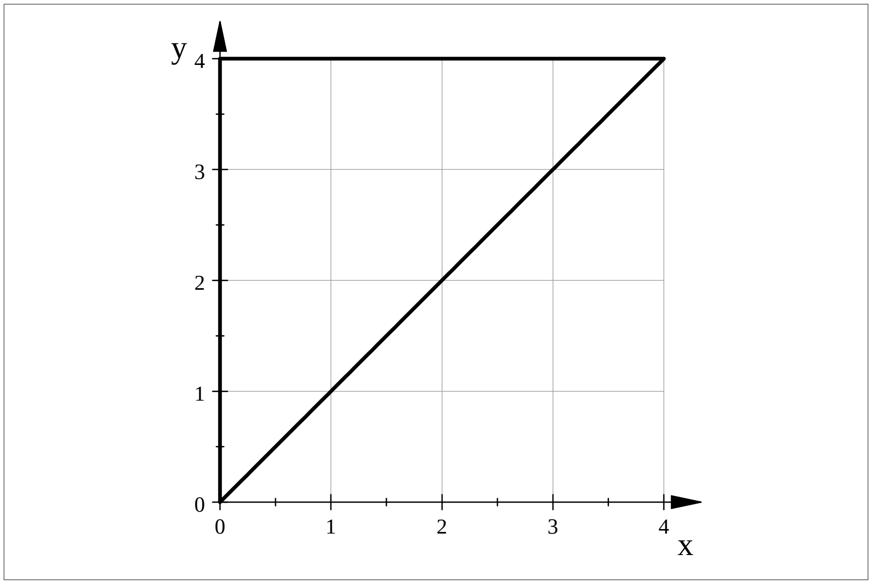
1 [25 **pts**.] Set up iterated integrals for both orders of integration for

$$\iint_D y^2 e^{xy} dA, \quad D \text{ is bounded by } y = x, y = 4, x = 0$$

Sketch D and evaluate this double integral.

Solution: The region D is shown below.

x



$$\iint_D y^2 e^{xy} dA = \int_0^4 \int_x^4 y^2 e^{xy} dy dx$$
$$= \int_0^4 \int_0^y y^2 e^{xy} dx dy$$

Evaluation of the first integral requires using integration by parts, whereas for the second integral we have

$$\iint_D y^2 e^{xy} dA = \int_0^4 \int_0^y y^2 e^{xy} dx dy = \int_0^4 \left[\frac{y^2 e^{xy}}{y} \right]_{x=0}^{x=y} dy$$
$$= \int_0^4 [ye^{xy}]_{x=0}^{x=y} dy = \int_0^4 \left[ye^{y^2} - y \right] dy$$
$$= \left[\frac{1}{2} e^{y^2} - \frac{1}{2} y^2 \right]_0^4 = \left(\frac{1}{2} e^{16} - 8 \right) - \left(\frac{1}{2} - 0 \right) = \frac{1}{2} e^{16} - \frac{17}{2}$$

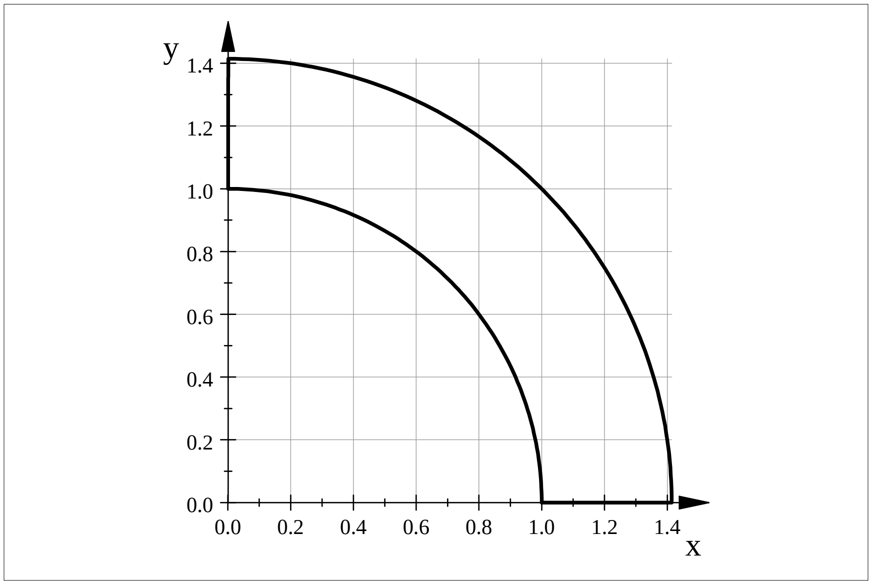
2 a $[$20 **pts**.$]$ Evaluate

$$\iint_R x dA$$

where R is the region in the first quadrant that lies between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 2$. Sketch R.

Solution: The region R is shown below.

1



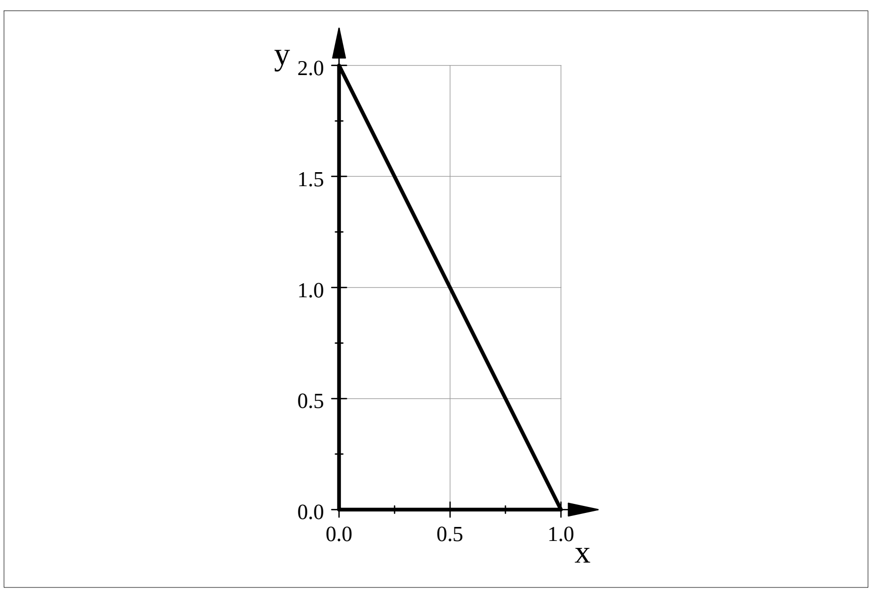
Then

$$\iint_R x dA = \int_0^{\frac{\pi}{2}} \int_1^{\sqrt{2}} (r\cos\theta) r dr d\theta$$

$$= \int_0^{\frac{\pi}{2}} \cos\theta \left[\frac{r^3}{3} \right]_1^{\sqrt{2}} d\theta = \frac{1}{3}\left(2^{\frac{3}{2}} - 1\right)[\sin\theta]_0^{\frac{\pi}{2}} = \frac{1}{3}\left(2^{\frac{3}{2}} - 1\right)$$

2 b $\left[\right.$15 **pts**.$\left.\right]$ Give an integral for the surface area of the part of the surface $z = x^2 + y$ that lies above the triangle in the x, y –plane with vertices $(0,0), (1,0),$ and $(0,2)$. Sketch the triangle. DO NOT EVALUATE THIS INTEGRAL.

Solution: $(0,0,1,0,0,2,0,0)$



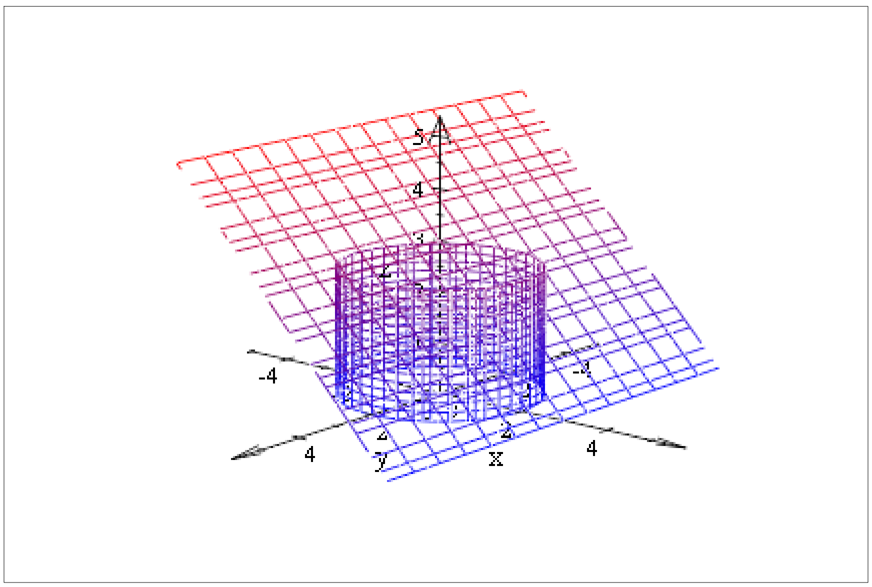
We need the equation of the hypotenuse of the triangle. The slope is $m = \frac{2-0}{0-2} = -2.$ Thus $y = mx + b = -2x + 2.$

Let R denote the region enclosed by the triangle. The surface area is given by

$$\begin{aligned} A(S) &= \iint_R \sqrt{1+(z_x)^2+(z_y)^2}\,dA \\ &= \iint_R \sqrt{1+(2x)^2+(1)^2}\,dA \\ &= \int_0^1\int_0^{2-2x} \sqrt{2+4x^2}\,dydx \\ &= \int_0^2\int_0^{\frac{y-2}{-2}} \sqrt{2+4x^2}\,dxdy \end{aligned}$$

3 [20 **pts**.] Use cylindrical coordinates to set up an iterated triple integral for the volume bounded by the cylinder $x^2+y^2=4$ and the planes $z=0$ and $y+z=3$. Evaluate this integral.

Solution:
$x^2+y^2=4$



z goes from 0 to the place which is $z=3-y=3-r\sin\theta$. r and θ range over the circle $x^2+y^2=4$ in the $x,y-plane$. Thus

$$\begin{aligned} V &= \int_0^{2\pi}\int_0^2\int_0^{3-r\sin\theta} rdzd\theta = \int_0^{2\pi}\int_0^2\left(3r-r^2\sin\theta\right)drd\theta \\ &= \int_0^{2\pi}\left(6-\frac{8}{3}\sin\theta\right)d\theta = \left[6\theta+\frac{8}{3}\cos\theta\right]_0^{2\pi} = 12\pi \end{aligned}$$

4 [20 **pts**.] Use spherical coordinates to evaluate

$$\int_{-2}^{2}\int_0^{\sqrt{4-y^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2\sqrt{x^2+y^2+z^2}\,dzdxdy$$

Solution: The region of integration is the solid hemisphere $x^2+y^2+z^2\le 4, x\ge 0$. Thus

$$
\begin{aligned}
\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2\sqrt{x^2+y^2+z^2}\,dzdxdy &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{\pi}\int_0^2 (\rho\sin\theta\sin\varphi)^2\sqrt{\rho^2}\,\rho^2\sin\varphi d\rho d\varphi d\theta \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2\theta d\theta\int_0^{\pi}\sin^3\varphi d\varphi\int_0^2\rho^5 d\rho \\
&= \left[-\frac{1}{2}\cos\theta\sin\theta+\frac{1}{2}\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[-\frac{1}{3}\sin^2\varphi\cos\varphi-\frac{2}{3}\cos\varphi\right]_0^{\pi}\left[\frac{1}{6}\rho^6\right] \\
&= \left[\frac{\pi}{2}\right]\left[\frac{2}{3}+\frac{2}{3}\right]\left[\frac{32}{3}\right] = \frac{64}{9}\pi
\end{aligned}
$$

Table of Integrals

$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \sin^3 x dx = -\frac{1}{3}\sin^2 x \cos x - \frac{2}{3}\cos x + C$
$\int \cos^3 x dx = \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x + C$
$\int t e^{at} dt = \frac{1}{a^2} e^{at}(at - 1) + C$
$\int t^2 e^{at} dt = \frac{1}{a^3} e^{at}\left(a^2 t^2 - 2at + 2\right) + C$