Ma 227 **Exam II A Solutions** **11/8/11**

Name: ______________________________

Lecture Section: ______

I pledge my honor that I have abided by the Stevens Honor System. ________________________________

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.
There is a table of integrals on the last page of the exam.

Score on Problem #1 ________

#2a ________

#2b ________

#3 ________

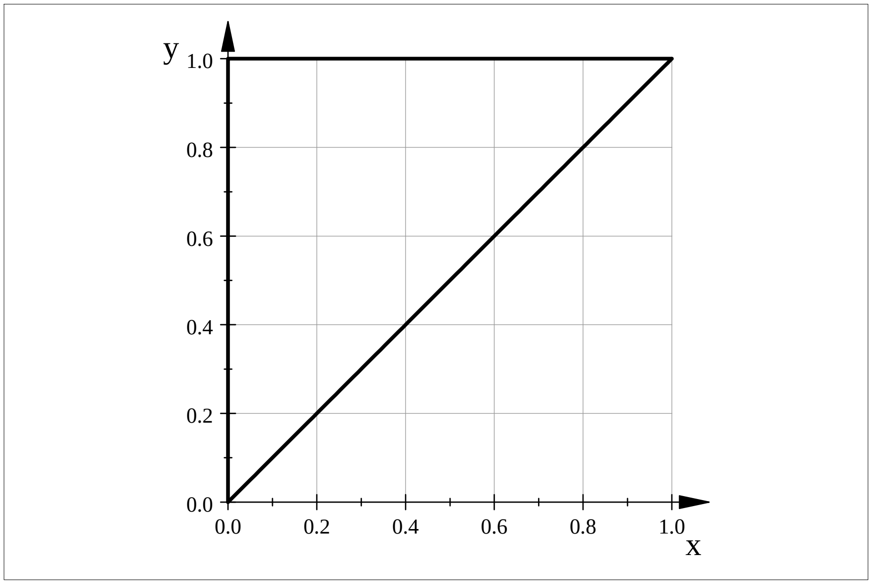
#4 ________

Total Score ________

1 [25 **pts**.] Evaluate

$$\int_0^1 \int_x^1 x\sqrt{1+y^3}\, dydx$$

Sketch the region of integration.
Solution: The region of integration is shown below
x



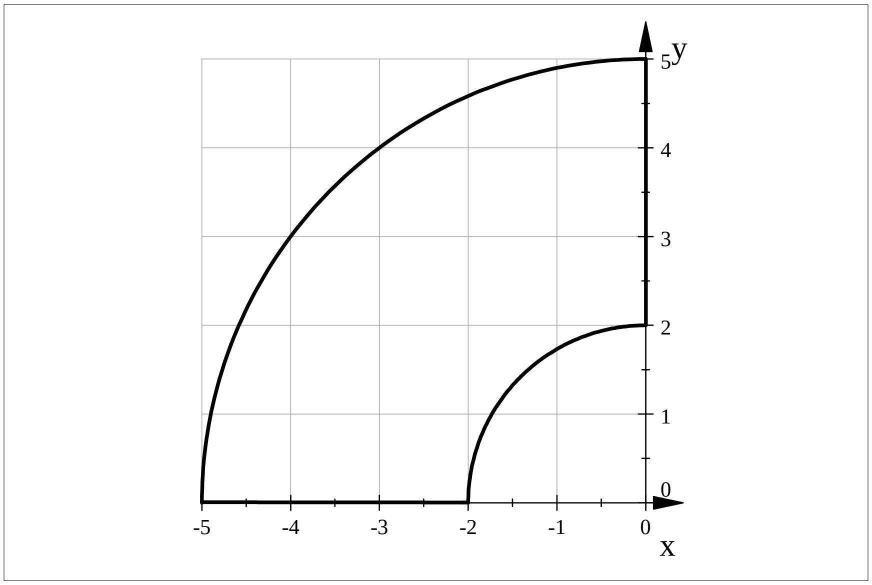
Thus

$$\begin{aligned}
\int_0^1 \int_x^1 x\sqrt{1+y^3}\, dydx &= \int_0^1 \int_0^y x\sqrt{1+y^3}\, dxdy \\
&= \int_0^1 \frac{x^2}{2}\Big|_0^y \sqrt{1+y^3}\, dy \\
&= \frac{1}{2}\int_0^1 y^2\sqrt{1+y^3}\, dy \\
&= \frac{1}{6}\left(\frac{2}{3}\right)\left(1+y^3\right)^{\frac{3}{2}}\Big|_0^1 \\
&= \frac{1}{9}\left(2^{\frac{3}{2}}-1\right)
\end{aligned}$$

2 a [20 **pts**.] Evaluate

$$\iint_R 2xydA$$

where R is the region in the second quadrant that lies between the circles of radius 2 and 5 centered at the origin. Sketch R.
Solution: The region R is shown below.



$\frac{\pi}{2} = 1.5708$

Thus

$$\iint_R 2xy\,dA = 2\int_{\frac{\pi}{2}}^{\pi}\int_2^5 (r\cos\theta)(r\sin\theta)r\,dr\,d\theta$$
$$= 2\int_{\frac{\pi}{2}}^{\pi}\left.\frac{r^4}{4}\right|_2^5 \sin\theta\cos\theta\,d\theta$$
$$= \frac{1}{2}(609)\left.\frac{\sin^2\theta}{2}\right|_{\frac{\pi}{2}}^{\pi} = \frac{609}{4}(0-1) = -\frac{609}{4}$$

2b $\left[15\ pts\right]$ Give an integral in polar coordinates for the surface area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane

$z = 9.$ DO NOT EVALUATE THIS INTEGRAL.

Solution: This is example 2 on page 871 of our text.

The plane intersects the paraboloid in the circle $x^2 + y^2 = 9,\ z = 9.$ Therefore, the given surface lies above the disk D with center the origin and radius 3.

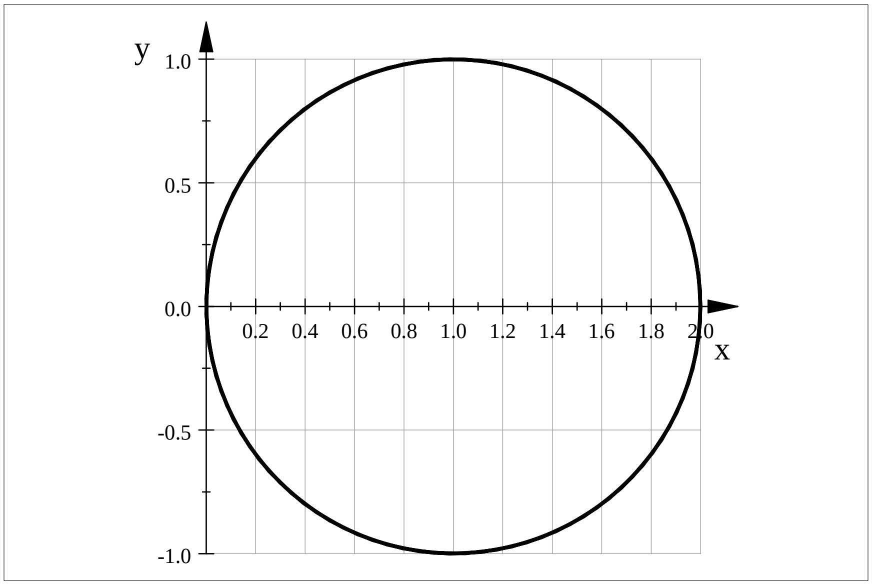
$$A = \iint_D \sqrt{1 + (z_x)^2 + (z_y)^2}\,dA$$
$$= \iint_D \sqrt{1 + (2x)^2 + (2y)^2}\,dA$$
$$= \iint_D \sqrt{1 + 4\left(x^2 + y^2\right)}\,dA$$
$$= \int_0^{2\pi}\int_0^3 \sqrt{1 + 4r^2}\,r\,dr\,d\theta$$

3 $\left[20\ \textbf{pts}.\right]$ Use cylindrical coordinates to set up an iterated triple integral for the volume of the solid

that lies under the paraboloid
$z = x^2 + y^2$, above the plane $z = 0$ and inside the cylinder $x^2 + y^2 = 2x$. DO NOT EVALUATE THIS INTEGRAL.

Solution: In cylindrical coordinates the equation of the paraboloid is $z = r^2$. The cylinder is given by $r^2 = 2r\cos\theta$ or $r = 2\cos\theta$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, since the circle has the equation $(x-1)^2 + y^2 = 1$ in rectangular coordinates and is centered at $(0,1)$, has radius 1 and is located in the first and fourth quadrants.

$2\cos\theta$



Thus

$$V = \iiint_E dV = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} \int_0^{r^2} r dz dr d\theta$$

The evaluation below is not part of the solution.

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} r^2 r dr d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(2\cos\theta)^4}{4} d\theta$$

$$= \frac{2^4}{4}\left(\frac{3}{8}\theta + \frac{1}{4}\sin 2\theta + \frac{1}{32}\sin 4\theta\right)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 4\left(\frac{3}{8}\right)(\pi) = \frac{3\pi}{2}$$

4 [20 **pts**.] Use spherical coordinates to evaluate

$$\iiint_E 16z dV$$

where E is the upper half of the sphere $x^2 + y^2 + z^2 = 1$.

Solution: Since we are taking the upper half of the sphere the limits for the variables are,

$$0 \le \rho \le 1$$
$$0 \le \theta \le 2\pi$$
$$0 \le \phi \le \frac{\pi}{2}$$

The integral is then

$$\begin{aligned}
\iiint\limits_E 16z\,dV &= \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \int_0^1 16(\rho\cos\phi)\rho^2 \sin\phi\, d\rho\, d\theta\, d\phi \\
&= 16 \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \int_0^1 \rho^3 \sin\phi\cos\phi\, d\rho\, d\theta\, d\phi \\
&= \frac{16}{4} \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \sin\phi\cos\phi\, d\theta\, d\phi \\
&= 4(2\pi)\frac{\sin^2\phi}{2}\Bigg|_0^{\frac{\pi}{2}} = 4\pi
\end{aligned}$$

Table of Integrals

$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \sin^3 x dx = -\frac{1}{3}\sin^2 x \cos x - \frac{2}{3}\cos x + C$
$\int \cos^3 x dx = \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x + C$
$\int \sin^4 x dx = \frac{3}{8}x - \frac{3}{16}\pi - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$
$\int \cos^4 x dx = \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$
$\int te^{at} dt = \frac{1}{a^2}e^{at}(at - 1) + C$
$\int t^2 e^{at} dt = \frac{1}{a^3}e^{at}\left(a^2t^2 - 2at + 2\right) + C$