Ma 227 **Exam II A** **11/13/12**

Name: ________________________________

Lecture Section: ________ **Recitation Section**: _______________

I pledge my honor that I have abided by the Stevens Honor System. ___________________________________

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Score on Problem #1a ________

#1b ________

#2________

#3a ________

#3b ________

#4 ________

Total Score ________

1a [20 **pts**.] Evaluate

$$\int_C xy^4 ds$$

where C is the right half of the circle, $x^2 + y^2 = 16$ traversed in the counter clockwise direction.

1b [**15 pts**.] Let

$$\vec{F}(x,y,z) = F_1(x,y,z)\vec{i} + F_2(x,y,z)\vec{j} + F_3(x,y,z)\vec{k}$$

Show that

$$\mathrm{div}\left(\mathrm{curl}\vec{F}\right) = \nabla \cdot \left(\nabla \times \vec{F}\right) = 0$$

2 [20 **pts**.] Find a function $\phi(x,y,z)$ such that

$$\nabla\phi = \vec{F}(x,y,z) = \left(2x\cos y - 2z^3\right)\vec{i} + \left(2ye^z - x^2\sin y\right)\vec{j} + \left(3 + y^2e^z - 6xz^2\right)\vec{k}$$

3a [15 **pts**.] Evaluate

$$\oint_C y^3 dx - x^3 dy$$

directly without using Green's Theorem, where C is the positively oriented circle of radius 2 centered at the origin.

3b [15 **pts**.] Evaluate the line integral in 3a, namely

$$\oint_C y^3 dx - x^3 dy$$

using Green’s Theorem.

4 [15] **pts**. Use Green's Theorem to find the area of the ellipse bounded by

$$\frac{x^2}{4} + \frac{y^2}{9} = 1.$$

Table of Integrals

$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \sin^2 x \cos^2 x dx = \frac{1}{8}x - \frac{1}{32}\sin 4x + C$
$\int \sin^3 x dx = -\frac{1}{3}\sin^2 x \cos x - \frac{2}{3}\cos x + C$
$\int \cos^3 x dx = \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x + C$
$\int \sin^4 x dx = \frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$
$\int \cos^4 x dx = \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$
$\int xe^x dx = e^x(x-1) + C$
$\int x^2 e^x dtx = e^x\left(x^2 - 2x + 2\right) + C$