Total Score

1a [20 pts.] Evaluate

$$\int_C \vec{F} \cdot d\vec{r}$$

where $\vec{F} = z\vec{i} + y^2\vec{j} + x\vec{k}$ and *C* is the curve given by $C: x(t) = t + 1, y(t) = e^t, z(t) = t^2 \quad 0 \le t \le 2$

$$C: x(t) = t + 1, y(t) = e^t, z(t) = t^2 \quad 0 \le t \le 2$$

1b [15 pts.] If the function f(x, y, z) has continuous second-order partial derivatives, show that $\operatorname{curl}(\operatorname{grad} f) = 0$

2 [20 **pts**.] Find a function
$$\phi(x, y, z)$$
 such that
$$\nabla \phi = \vec{F}(x, y, z) = (2xyz^{-1})\vec{i} + (z + x^2z^{-1})\vec{j} + (y - x^2yz^{-2})\vec{k}$$

$$\oint_C xy^2 dx + xdy$$

directly without using Green's Theorem, where C is the positively oriented circle of radius 1 centered at the origin.

3b [15 **pts**.] Evaluate the line integral in 3a, namely

$$\oint_C xy^2 dx + xdy$$

using Green's Theorem.

$$\oint_C (1 + \tan^5 x) dx + \left(x^2 + e^y\right) dy$$

Where C is the positively oriented boundary of the region R enclosed by the curves $y = \sqrt{x}$, x = 1, and y = 0. Be sure to sketch C.

Table of Integrals

$$\int \sin^2 x dx = -\frac{1}{2} \cos x \sin x + \frac{1}{2} x + C$$

$$\int \cos^2 x dx = \frac{1}{2} \cos x \sin x + \frac{1}{2} x + C$$

$$\int \sin^3 x dx = -\frac{1}{3} \sin^2 x \cos x - \frac{2}{3} \cos x + C$$

$$\int \cos^3 x dx = \frac{1}{3} \cos^2 x \sin x + \frac{2}{3} \sin x + C$$

$$\int te^t dt = e^t (t - 1) + C$$

$$\int t^2 e^t dt = e^t (t^2 - 2t + 2) + C$$