# Ma 227 Exam II B
# Solutions 11/8/11

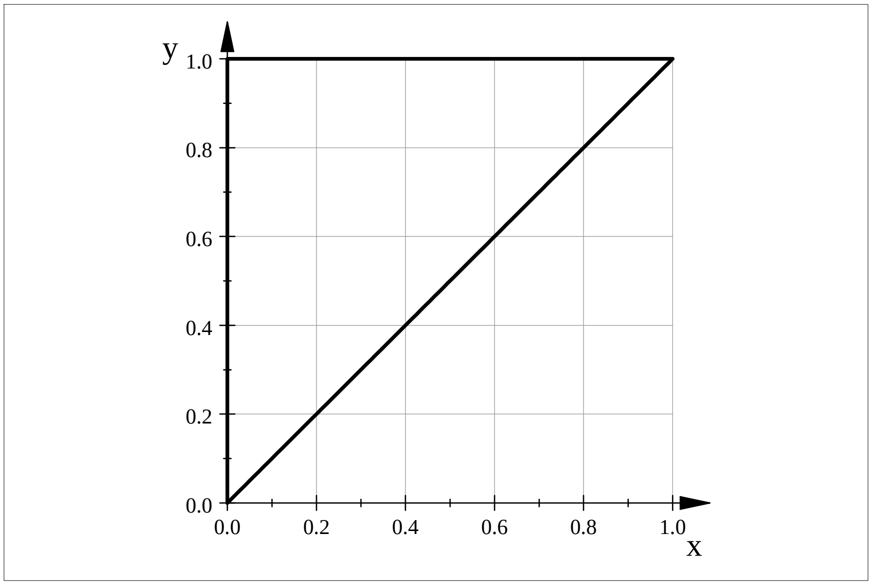
**Name**: ____________________________

**Lecture Section**: ______

**1** [25 **pts**.] Evaluate

$$\int_0^1 \int_x^1 x^2 \sqrt{1+y^4}\, dydx$$

Sketch the region of integration.
Solution: The region of integration is shown below
*x*




Thus

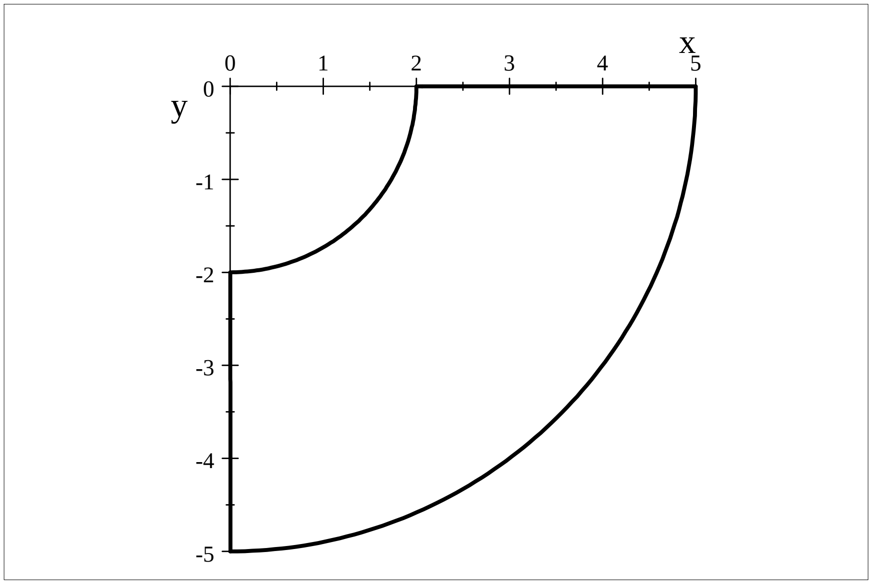
$$\begin{aligned}
\int_0^1 \int_x^1 x^2 \sqrt{1+y^4}\, dydx &= \int_0^1 \int_0^y x^2 \sqrt{1+y^4}\, dxdy \\
&= \int_0^1 \frac{x^3}{3} \Big|_0^y \sqrt{1+y^4}\, dy \\
&= \frac{1}{3} \int_0^1 y^3 \sqrt{1+y^4}\, dy \\
&= \frac{1}{3}\left(\frac{1}{4}\right)\left(\frac{2}{3}\right)(1+y^4)^{\frac{3}{2}} \Big|_0^1 \\
&= \frac{1}{18}\left(2^{\frac{3}{2}} - 1\right)
\end{aligned}$$

**2 a** [20 **pts.**] Evaluate

$$\iint_R xy^2 dA$$

where $R$ is the region in the fourth quadrant that lies between the circles of radius 2 and 5 centered at the origin. Sketch $R$.

Solution: The region $R$ is shown below.




$\frac{\pi}{2} = 1.5708$

Thus

$$\iint_R xy^2 dA = \int_{\frac{3\pi}{2}}^{2\pi} \int_2^5 (r\cos\theta)(r\sin\theta)^2 r dr d\theta$$

$$= \int_{\frac{3\pi}{2}}^{2\pi} \frac{r^5}{5}\Big|_2^5 \sin^2\theta\cos\theta d\theta$$

$$= \frac{1}{5}(5^5 - 2^5)\frac{\sin^3\theta}{3}\Big|_{\frac{3\pi}{2}}^{2\pi} = \frac{(5^5 - 2^5)}{15}\left(0 - (-1)^3\right) = \frac{(5^5 - 2^5)}{15} = \frac{3093}{15}$$

**2b** [15 *pts*] Give an integral in polar coordinates for the surface area of the part of the paraboloid $z = 2x^2 + 2y^2$ that lies under the plane
$z = 18$. DO NOT EVALUATE THIS INTEGRAL.
Solution: This is example 2 on page 871 of our text, slightly modified.

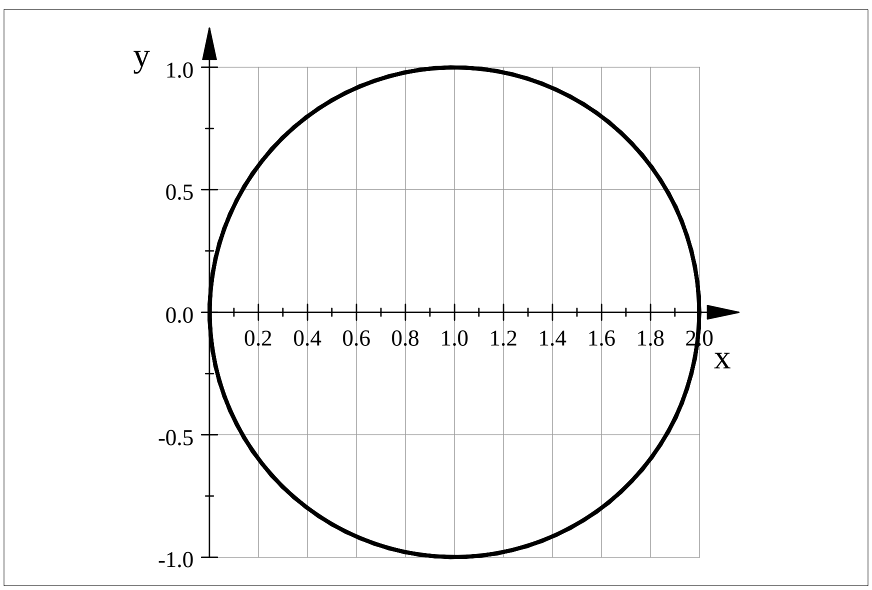
The plane intersects the paraboloid in the circle $x^2 + y^2 = 9$, $z = 9$. Therefore, the given surface lies above the disk $D$ with center the origin and radius 3.

$$\begin{aligned} A &= \iint_D \sqrt{1 + (z_x)^2 + (z_y)^2}\, dA \\ &= \iint_D \sqrt{1 + (4x)^2 + (4y)^2}\, dA \\ &= \iint_D \sqrt{1 + 16(x^2 + y^2)}\, dA \\ &= \int_0^{2\pi} \int_0^3 \sqrt{1 + 16r^2}\, r dr d\theta \end{aligned}$$

**3** [20 **pts**.] Use cylindrical coordinates to set up an iterated triple integral for the volume of the solid that lies under the paraboloid
$z = 4 - x^2 4y^2$, above the plane $z = 0$ and inside the cylinder $x^2 + y^2 = 2x$. DO NOT EVALUATE THIS INTEGRAL.

Solution: In cylindrical coordinates the equation of the paraboloid is $z = 4 - r^2$. The cylinder is given by $r^2 = 2r\cos\theta$ or $r = 2\cos\theta$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, since the circle has the equation $(x-1)^2 + y^2 = 1$ in rectangular coordinates and is centered at $(0, 1)$, has radius 1 and is located in the first and fourth quadrants.

$2\cos\theta$




Thus

$$V = \iiint\limits_E dV = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} \int_0^{4-r^2} r\,dz\,dr\,d\theta$$

**4** [20 **pts**.] Use spherical coordinates to evaluate

$$\iiint\limits_E 16X\,dV$$

where $E$ is the portion of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant.

Solution: Since we are taking the portion of the sphere in the first octant, the limits for the variables are,

$$0 \le \rho \le 1$$
$$0 \le \theta \le \frac{\pi}{2}$$
$$0 \le \phi \le \frac{\pi}{2}$$

The integral is then

$$\begin{aligned}
\iiint\limits_E 16x\,dV &= \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^1 16(\rho\cos\theta\sin\phi)\rho^2\sin\phi\, d\rho\, d\theta\, d\phi \\
&= 16\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^1 \rho^3\sin^2\phi\cos\theta\, d\rho\, d\theta\, d\phi \\
&= \frac{16}{4}\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin^2\phi\cos\theta\, d\theta\, d\phi \\
&= 4\int_0^{\frac{\pi}{2}} \sin\theta\Big|_{\theta=0}^{\theta=\frac{\pi}{2}} \sin^2\phi\, d\phi \\
&= 4\int_0^{\frac{\pi}{2}} \sin^2\phi\, d\phi \\
&= 4\left[-\frac{1}{2}\cos\phi\sin\phi + \frac{1}{2}\phi\right]_{\phi=0}^{\phi=\frac{\pi}{2}} \\
&= 4\left[\frac{\pi}{4}\right] = \pi
\end{aligned}$$

The last integration is obtained from the table of integrals at the back of the exam.

# Table of Integrals

| |
|---|
| $\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$ |
| $\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$ |
| $\int \sin^3 x dx = -\frac{1}{3}\sin^2 x \cos x - \frac{2}{3}\cos x + C$ |
| $\int \cos^3 x dx = \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x + C$ |
| $\int \sin^4 x dx = \frac{3}{8}x - \frac{3}{16}\pi - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$ |
| $\int \cos^4 x dx = \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$ |
| $\int t e^{at} dt = \frac{1}{a^2}e^{at}(at - 1) + C$ |
| $\int t^2 e^{at} dt = \frac{1}{a^3}e^{at}(a^2t^2 - 2at + 2) + C$ |