

**Ma 227**

**Exam III**

**12/5/07**

Name: \_\_\_\_\_

Lecture Section: \_\_\_\_\_

Recitation Section: \_\_\_\_\_

*I pledge my honor that I have abided by the Stevens Honor System.* \_\_\_\_\_

**You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.**

Score on Problem #1a \_\_\_\_\_

#1b \_\_\_\_\_

#2 \_\_\_\_\_

#3a \_\_\_\_\_

#3b \_\_\_\_\_

#4a \_\_\_\_\_

#4b \_\_\_\_\_

Total Score \_\_\_\_\_

**1a [15 pts.]** Evaluate the line integral

$$\int_C y^3 dx + x^2 dy$$

where  $C$  is the arc of the parabola  $x = 1 - y^2$  from  $(0, -1)$  to  $(0, 1)$ .

**1b [15 pts.]** Let  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$  and  $r = |\vec{r}|$ . Show that

$$\nabla(\ln r) = \frac{\vec{r}}{r^2}$$

2 [20 pts.] The vector function

$$\vec{F} = e^y \vec{i} + (xe^y + e^z) \vec{j} + ye^z \vec{k}$$

is conservative. Evaluate  $\int_C \vec{F} \cdot d\vec{r}$  where  $C$  is the line segment from  $(0, 2, 0)$  to  $(4, 0, 3)$ .

**3a** [15 pts.] Evaluate

$$\oint_C xy^2 dx - x^2 y dy$$

*directly without using Green's Theorem*, where  $C$  consists of the parabola  $y = x^2$  from  $(-1, 1)$  to  $(1, 1)$  and the line segment from  $(1, 1)$  to  $(-1, 1)$ . Sketch  $C$ .

**3b** [15 pts.] Evaluate the line integral in 3a by using Green's Theorem.

**4 a [5 pts.]** Let  $S$  be the part of the paraboloid  $z = x^2 + y^2 - 3$  below the plane  $z = 1$ . Sketch  $S$  and give a parametrization of  $S$  in rectangular coordinates.

**4 b [15 pts.]** Give an expression for

$$\iint_S (y + z) dS$$

in rectangular coordinates where  $S$  is the surface in part 4a. Do *not* evaluate your expression.