Ma 227 **Exam III** **12/6/10**

Name: ________________________________

Lecture Section: ________ **Recitation Section**: _______________

I pledge my honor that I have abided by the Stevens Honor System. ___________________________________

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Score on Problem #1a ________

#1b ________

#2 ________

#3a ________

#3b ________

#4a ________

#4b ________

Total Score ________

1a [**15 pts**.] Evaluate the line integral

$$\int_C \vec{F} \cdot d\vec{r}$$

where $\vec{F} = \sqrt{xy}\,\vec{i} + e^y\vec{j} + xz\vec{k}$ and C is the curve given by $\vec{r}(t) = t^4\vec{i} + t^2\vec{j} + t^3\vec{k},\ 0 \le t \le 1.$

1b [**15 pts**.] Let $\vec{A}(x, y, z) = 2z\vec{i} + e^x\vec{j} - e^{-y}\vec{k}.$ Calculate $\nabla \cdot \left(\nabla \times \vec{A}\right)$ directly.

2 a [12 **pts**.] Find a function $\phi(x,y,z)$ such that

$$\nabla\phi = \vec{F}(x,y,z) = (2x - y - z)\vec{i} + (2y - x)\vec{j} + (2z - x)\vec{k}$$

2 b [8 **pts**.]   Evaluate

$$\int_C \vec{F} \cdot d\vec{r}$$

where $\vec{F}$ is the vector field in part 2a) and C is the curve given by the vector equation

$$\vec{r}(t) = \left(1 + 2t^2\right)\vec{i} + (1 - t^5)\vec{j} + \left(1 + 2t^6\right)\vec{k} \qquad 0 \le t \le 1$$

3a [15 **pts**.] Evaluate

$$\oint_C y\,dx + \left(x + y^2\right)dy$$

directly without using Green's Theorem, where C is the ellipse $4x^2 + 9y^2 = 36$ with counterclockwise orientation. (Hint: A parametrization of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $x = a\cos t,\ y = b\sin t$.)

3b $\left[10\ \textbf{pts.} \right]$ Evaluate the line integral in 3a, namely

$$\oint_C y\,dx + \left(x + y^2\right) dy$$

using Green's Theorem.

4*a* [10 **pts**] Parametrize the surface S that is the part of the paraboloid

$$y = x^2 + z^2$$

that lies between the planes $y = 4$ and $y = 0.$ Sketch the surface S.

4*b* [15 **pts**.] Give an iterated double integral equal to

$$\iint_S xz dS$$

where S is the surface in part 3a. Do *not* evaluate your expression.

Table of Integrals

$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \sin^3 x dx = -\frac{1}{3}\sin^2 x \cos x - \frac{2}{3}\cos x + C$
$\int \cos^3 x dx = \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x + C$
$\int te^t dt = e^t(t-1) + C$
$\int t^2 e^t dt = e^t\left(t^2 - 2t + 2\right) + C$