**Ma 227** **Exam III A** **12/6/11**

**Name**: ________________________________

**Lecture Section**: ________ **Recitation Section**: _______________

*I pledge my honor that I have abided by the Stevens Honor System.* ___________________________________

**You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.**

Score on Problem #1a ________

#1b ________

#2a ________

#2b ________

#3a ________

#3b ________

#4a ________

#4b ________

Total Score ________

**1a** [**15 pts**.] Evaluate the line integral

$$\int_C \vec{F} \cdot d\vec{r}$$

where $\vec{F}(x,y,z) = y\vec{i} + x\vec{j} + z^2\vec{k}$ and $C$ is the curve given by $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + t\vec{k},\ 0 \le t \le 2\pi$.

**1b** [**15 pts**.] Let $g(x,y,z)$ be a scalar function whose cross partial derivatives are continuous. Calculate $\nabla \times \nabla g$.

**2 a** [14 **pts**.]  Find a function $\phi(x,y,z)$ such that

$$\nabla\phi = \vec{F}(x,y,z) = 2xy^3z^4\vec{i} + \left(3x^2y^2z^4 + y\right)\vec{j} + \left(4x^2y^3z^3 + 2z\right)\vec{k}$$

**2 b** [6 **pts**.]    Let Γ be any closed curve in three space.  What is the value of $\oint_\Gamma \vec{F} \cdot d\vec{r}$ where $\vec{F}$ is the vector function in 2a.  Justify your conclusion.

**3a** [15 **pts**.] Evaluate

$$\oint_C xydx + x^2y^3dy$$

*directly without using Green's Theorem*, where $C$ is the triangle with vertices $(0,0),(1,0),(1,2)$.

**3b** [10 **pts**.] Evaluate the line integral in 3a, namely

$$\oint_C xydx + x^2y^3dy$$

using Green's Theorem.

4*a* [10 **pts**] Let $S$ be the upper half of the sphere of radius 2 centered at the origin. Sketch and parametrize the surface $S$. Be sure to include the range of the parameters.

4*b* [15 **pts**.] Give an iterated double integral equal to

$$\iint_S z\,dS$$

where $S$ is the surface in part 4a. Do *not* evaluate your expression.

# Table of Integrals

| |
|---|
| $\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$ |
| $\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$ |
| $\int \sin^3 x dx = -\frac{1}{3}\sin^2 x \cos x - \frac{2}{3}\cos x + C$ |
| $\int \cos^3 x dx = \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x + C$ |
| $\int te^t dt = e^t(t-1) + C$ |
| $\int t^2 e^t dt = e^t\left(t^2 - 2t + 2\right) + C$ |