**Ma 227** **Exam III A** **12/3/13**

**Name**: ________________________________

**Lecture Section**: ________

I pledge my honor that I have abided by the Stevens Honor System.

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**You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.**

Score on Problem #1 ________

#2 ________

#3 ________

#4 _________

Total Score ________

**1** [30 **pts**.] Evaluate the surface integral

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n}\, dS$$

where

$$\vec{F} = x\vec{k}$$

and $S$ is the surface with parametrization

$$x = u^2 \quad y = v \quad z = u^3 - v^2 \quad 0 \le u \le 1 \quad 0 \le v \le 1$$

and oriented by an upward pointing normal.

**2** [20 **pts**.] Let $S$ be any closed bounded surface with outward directed normal $\vec{n}$ in $x, y, z$ –space, and

$$\vec{F} = \left(z^2 + xy^2\right)\vec{i} + \cos(x+z)\vec{j} + \left(e^{-y} - zy^2\right)\vec{k}.$$

Evaluate

$$\iint_S \vec{F} \cdot \vec{n}\, dS.$$

**3** [20 **pts**.] Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 2 & -4 \\ -1 & -1 \end{bmatrix}.$$

**4** [30 **pts**.] Consider the vector $\vec{F} = -y\vec{i} + 2x\vec{j} + (x+z)\vec{k}$ and let $S$ be the upper hemisphere

$$x^2 + y^2 + z^2 = 1, \quad z \geq 0$$

with an upward normal.

Note:

$$\text{curl}\vec{F} = 0\vec{i} - \vec{j} + 3\vec{k}$$

Using spherical coordinates since $\rho = 1$, a parametrization of the hemisphere is

$$x = \sin\phi\cos\theta, \qquad y = \sin\phi\sin\theta, \qquad z = \cos\phi$$

With this parametrization

$$\vec{r}(\phi,\theta) = \sin\phi\cos\theta\,\vec{i} + \sin\phi\sin\theta\,\vec{j} + \cos\phi\,\vec{k}$$

and a normal to the hemisphere is

$$\vec{r}_\phi \times \vec{r}_\theta = \sin^2\phi\cos\theta\vec{i} + \sin^2\phi\sin\theta\vec{j} + \sin\phi\cos\phi\vec{k}.$$

Show this normal is upward, and use this information to verify Stokes' Theorem.

# Table of Integrals

| |
|---|
| $\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$ |
| $\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$ |
| $\int \sin^3 x dx = -\frac{1}{3}\sin^2 x \cos x - \frac{2}{3}\cos x + C$ |
| $\int \cos^3 x dx = \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x + C$ |
| $\int te^t dt = e^t(t-1) + C$ |
| $\int t^2 e^t dt = e^t\left(t^2 - 2t + 2\right) + C$ |