**Total Score** 

1 [25 pts.] Find the eigenvalues and eigenvectors of the matrix

$$A = \left[ \begin{array}{cc} 6 & -3 \\ 2 & 1 \end{array} \right]$$

2 [25 pts.] The eigenvalues and eigenvectors of the matrix

$$A = \left[ \begin{array}{cc} 1 & -4 \\ 1 & 1 \end{array} \right]$$

are 
$$1 + 2i \leftrightarrow \begin{bmatrix} 2i \\ 1 \end{bmatrix}$$
 and  $1 - 2i \leftrightarrow \begin{bmatrix} -2i \\ 1 \end{bmatrix}$ .

Give a [real] general solution for the system

$$x' = Ax$$
.

3 The eigenvalues and eigenvectors of the matrix

$$B = \left[ \begin{array}{cc} 1 & 4 \\ 1 & 1 \end{array} \right]$$

are 
$$3 \leftrightarrow \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$
 and  $-1 \leftrightarrow \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ .

**3a** [20 pts.] Find a [particular] solution,  $x_p$ , to

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 6t \\ -1 \end{bmatrix}$$

**3b** [10 **pts**.] Give a general solution to

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 6t \\ -1 \end{bmatrix}$$

## [20 **pts**.] Rewrite the equation

$$y''' + t^2y'' + e^ty' + 3y = \sin 2t, \ y(0) = 2, y'(0) = -1, y''(0) = 0$$

as a system of differential equations in normal form with appropriate initial conditions.