Name: $\qquad$
Lecture Section: $\qquad$ Recitation Section: $\qquad$

I pledge my honor that I have abided by the Stevens Honor System.

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Score on Problem \#1 $\qquad$
$\qquad$
\#2
\#3a $\qquad$
\#3b $\qquad$
\#4 $\qquad$

Total Score
$\mathbf{1}$ [25 pts.] Find the eigenvalues and eigenvectors of the matrix

$$
A=\left[\begin{array}{cc}
6 & 3 \\
-2 & 1
\end{array}\right]
$$

2 [25 pts.] The eigenvalues and eigenvectors of the matrix

$$
A=\left[\begin{array}{cc}
1 & -9 \\
1 & 1
\end{array}\right]
$$

are $1+3 i \leftrightarrow\left[\begin{array}{c}3 i \\ 1\end{array}\right]$ and $1-3 i \leftrightarrow\left[\begin{array}{c}-3 i \\ 1\end{array}\right]$.
Give a [real] general solution for the system

$$
x^{\prime}=A x .
$$

3 The eigenvalues and eigenvectors of the matrix

$$
B=\left[\begin{array}{ll}
1 & 9 \\
1 & 1
\end{array}\right]
$$

are $4 \leftrightarrow\left[\begin{array}{l}3 \\ 1\end{array}\right]$ and $-2 \leftrightarrow\left[\begin{array}{c}-3 \\ 1\end{array}\right]$.
3a [20 pts.] Find a [particular] solution, $x_{p}$, to

$$
\left[\begin{array}{l}
x_{1}^{\prime} \\
x_{2}^{\prime}
\end{array}\right]=\left[\begin{array}{ll}
1 & 9 \\
1 & 1
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2}
\end{array}\right]+\left[\begin{array}{c}
16 t \\
4
\end{array}\right]
$$

3b [10 pts.] Give a general solution to

$$
\left[\begin{array}{l}
x_{1}^{\prime} \\
x_{2}^{\prime}
\end{array}\right]=\left[\begin{array}{ll}
1 & 9 \\
1 & 1
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2}
\end{array}\right]+\left[\begin{array}{c}
16 t \\
4
\end{array}\right]
$$

4 [20 pts.] Rewrite the equation

$$
y^{\prime \prime \prime}+t^{2} y^{\prime \prime}+e^{t} y^{\prime}+3 y=-\cos 4 t, y(0)=-2, y^{\prime}(0)=1, y^{\prime \prime}(0)=0
$$

as a system of differential equations in normal form with appropriate initial conditions.

