**Ma 227** **Final Exam Solutions** **5/10/05**

**Name**: ______________________________

**Lecture Section**: ______________________________

I pledge my honor that I have abided by the Stevens Honor System.

___________________________________________

**You may not use a calculator**, **cell phone**, **or computer while taking this exam**. **All work must be shown to obtain full credit**. **Credit will not be given for work not reasonably supported**. **When you finish**, **be sure to sign the pledge**.

**Directions**: Answer all questions. The point value of each problem is indicated. If you need more work space, continue the problem you are doing on the **other side of the page it is on**.

**There is a table of integrals at the end of the exam**.

Score on Problem #1 ________

#2 ________

#3 ________

#4 ________

#5 ________

#6 ________

#7 ________

#8 ________

Total ________

## Problem 1

**a**) (**10 points**)

Show that the vector field

$$\vec{F}(x,y,z) = x\vec{i} + e^y \sin z\vec{j} + e^y \cos z\vec{k}$$

is conservative.

Solution:

$$\nabla \times \vec{F} = \left| \begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & e^y \sin z & e^y \cos z \end{array} \right| \begin{array}{cc} \vec{i} & \vec{j} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ x & e^y \sin z \end{array} = e^y \cos z\vec{i} + 0\vec{j} + 0\vec{k} - 0\vec{k} - e^y \cos z\vec{i} - 0\vec{j} = 0$$

Therefore since $curl\vec{F} = 0$, the given force field is conservative.

**b**) (**15 points**)

Find a function $\phi(x,y,z)$ such that $\nabla\phi = \vec{F}$ where $\vec{F}$ is the vector field in 1a above. What can you say about

$$\oint_C \vec{F} \cdot d\vec{r}$$

where $C$ is any closed curve?

Solution: $\phi_x = x$ so

$$\phi = \frac{x^2}{2} + g(y,z)$$

Thus

$$\phi_y = \frac{\partial g}{\partial y} = e^y \sin z$$

so that

$$g = e^y \sin z + h(z)$$

and

$$\phi = \frac{x^2}{2} + e^y \sin z + h(z)$$

Then

$$\phi_z = e^y \cos z + h'(z) = e^y \cos z$$

so $h'(z) = 0$ and $h(z) = K$. Thus

$$\phi = \frac{x^2}{2} + e^y \sin z + K$$

Since the force field is conservative then $\oint_C \vec{F} \cdot d\vec{r} = 0$, where $C$ is any closed curve.

## Problem 2

**a**) (**12 points**)

Give an expression in cylindrical coordinates for the

$$\iiint_E y dV$$

where E is the region that lies below the plane $z = x + 2$ above the $x,y$-plane and between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ . Do *not* evaluate this integral.

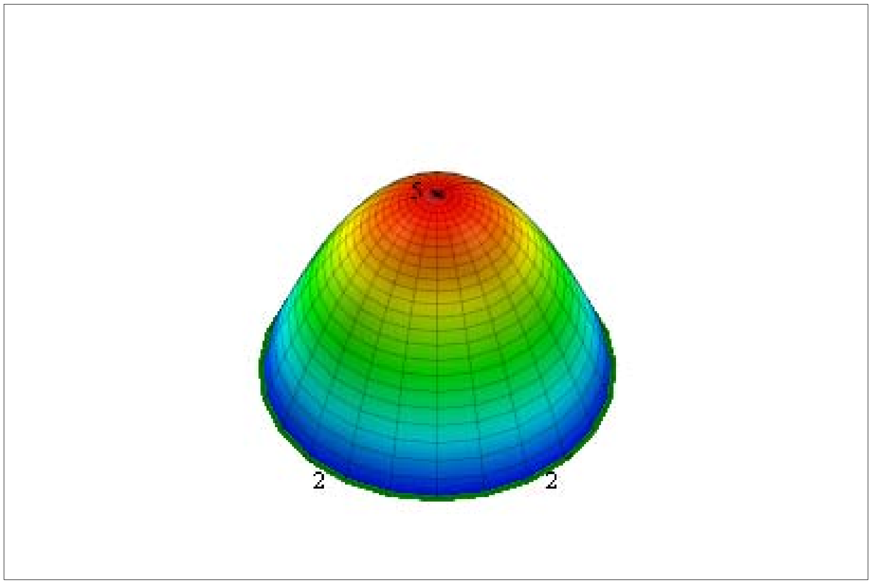
Solution: In cylindrical coordinates $x = r\cos\theta, y = r\sin\theta, z = z.$ Thus $z = 2 + r\cos\theta$, $r$ goes from 1 to 2 and $0 \leq \theta \leq 2\pi$. Hence

$$\iiint_E y\,dV = \int_0^{2\pi}\int_1^2\int_0^{2+r\cos\theta}(r\sin\theta)r\,dz\,dr\,d\theta$$

**b**) (**13 points**)

Use Stokes' Theorem to evaluate $\iint_S (\nabla \times \vec{F}) \cdot \vec{n}\,dS$ where $\vec{F} = z^2\vec{i} - 3x\vec{j} + x^3y^3\vec{k}$ and $S$ is the part of the surface $z = 5 - x^2 - y^2$ above the plane $z = 1.$ Assume that $S$ oriented upwards. Sketch $S$.

Solution: $(r, \theta, 5 - r^2)$




Stokes' Theorem is

$$\iint_S (\nabla \times \vec{F}) \cdot \vec{n}\,dS = \oint_C \vec{F} \cdot d\vec{r}$$

Now the boundary $C$ of $S$ will be where the surface intersects $z = 1,$ that is, when $1 = 5 - x^2 - y^2$ or $x^2 + y^2 = 4.$ Thus

$$C:\ \ x = 2\cos t, y = 2\sin t;\ \ 0 \leq t \leq 2\pi;\ z = 1$$

and

$\vec{F} = z^2\vec{i} - 3x\vec{j} + x^3y^3\vec{k}$

$$\vec{r}(t) = 2\cos t\vec{i} + 2\sin t\vec{j} + \vec{k}$$
$$\vec{F}(t) = (1)^2\vec{i} - 3(2)\cos t\vec{j} + (2\cos t)^3(2\sin t)^3\vec{k}$$

Then

$$\vec{r}'(t) = -2\sin t\vec{i} + 2\cos t\vec{j}$$

and

$$\vec{F}(t) \cdot \vec{r}'(t) = -2\sin t - 12\cos^2 t$$

Thus

$$\oint_C \vec{F}\cdot d\vec{r} = \int_0^{2\pi} \vec{F}(t)\cdot\vec{r}'(t)dt = \int_0^{2\pi}\left(-2\sin t - 12\cos^2 t\right)dt = [2\cos t - 6(\cos t\sin t + t)]_0^{2\pi} = -12\pi$$

## Problem 3

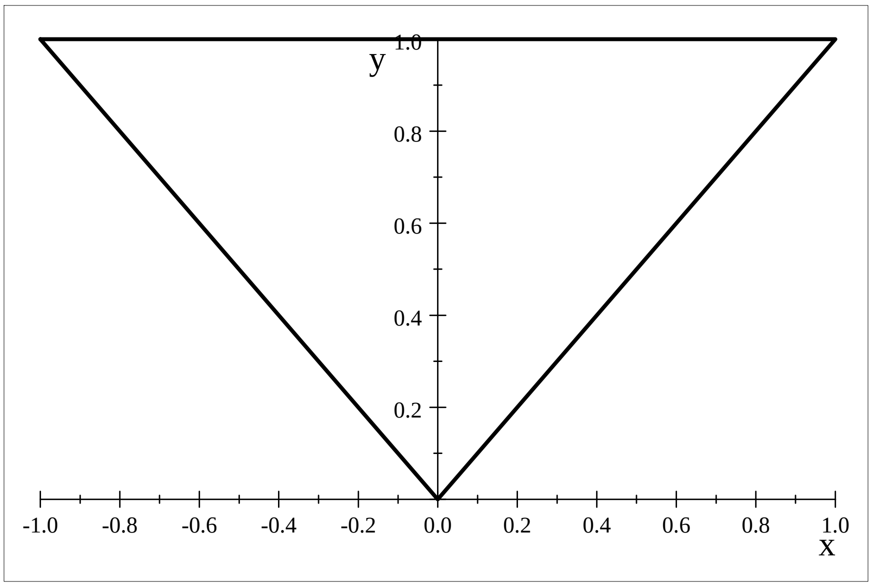
**a) (12 points)**

Find two iterated integrals representing

$$\iint_R y\,dA$$

where $R$ is the triangular region with vertices at $(-1,1),(0,0)$, and $(1,1)$. Sketch $R$. Do *not* evaluate these integrals.

Solution: $(-1,1,0,0,1,1,-1,1)$




Thus

$$\iint_R y\,dA = \int_0^1\int_{-y}^{y} y\,dx\,dy = \int_{-1}^{0}\int_{-x}^{1} y\,dy\,dx + \int_0^1\int_x^1 y\,dy\,dx$$

**b) (13 points)**

Find the surface area of the part of the plane $z = 8x + 4y$ that lies inside the cylinder $x^2 + y^2 = 16$

Solution: Since $z_x = 8, z_y = 4$

$$\text{Surface Area} = \iint_S dS = \iint_R \sqrt{1 + z_x^2 + z_y^2}\,dA = \iint_{x^2+y^2\le 16} \sqrt{1 + (8)^2 + (4)^2}\,dA$$

$$= 9\iint_{x^2+y^2\le 16} dA = 9\pi(4)^2 = 144\pi$$

## Problem 4

**a) (15 points)**

Verify Green's theorem for the line integral

$$\oint_C (x+y)dx + (x-y)dy$$

where $C$ is the positively oriented unit circle centered at the origin.
Solution: A parametrization of $C$ is $x = \cos t, y = \sin t \;\; 0 \le t \le 2\pi$. Thus

$$\oint_C (x+y)dx + (x-y)dy = \int_0^{2\pi} [(\cos t + \sin t)(-\sin t) + (\cos t - \sin t)(\cos t)]dt$$

$$= \int_0^{2\pi} \left(-2\sin t\cos t - \sin^2 t + \cos^2 t\right)dt = \left[-2\sin^2 t \;+ \frac{1}{2}\cos t\sin t - \frac{1}{2}t + \frac{1}{2}\cos t\sin t + \frac{1}{2}t\right]_0^{2\pi}$$

Here $P = x+y$ and $Q = x-y$ so

$$\iint_{x^2+y^2\le 1} (P_y - Q_x)dA = \iint_{x^2+y^2\le 1} [1-1]dA = 0$$

**b)** (**10 points**)
Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}.$$

Solution:

$$\begin{bmatrix} 2 & 3 & 1 & 0 \\ 2 & 2 & 0 & 1 \end{bmatrix} \to_{-R_1+R_2} \begin{bmatrix} 2 & 3 & 1 & 0 \\ 0 & -1 & -1 & 1 \end{bmatrix} \to_{3R_2+R_1} \begin{bmatrix} 2 & 0 & -2 & 3 \\ 0 & -1 & -1 & 1 \end{bmatrix} \to_{\frac{1}{2}R_1:(-1)R_2} \begin{bmatrix} 1 & 0 & -1 & \frac{3}{2} \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

Thus

$$A^{-1} = \begin{bmatrix} -1 & \frac{3}{2} \\ 1 & -1 \end{bmatrix}$$

## Problem 5

**a)** (**13 points**)

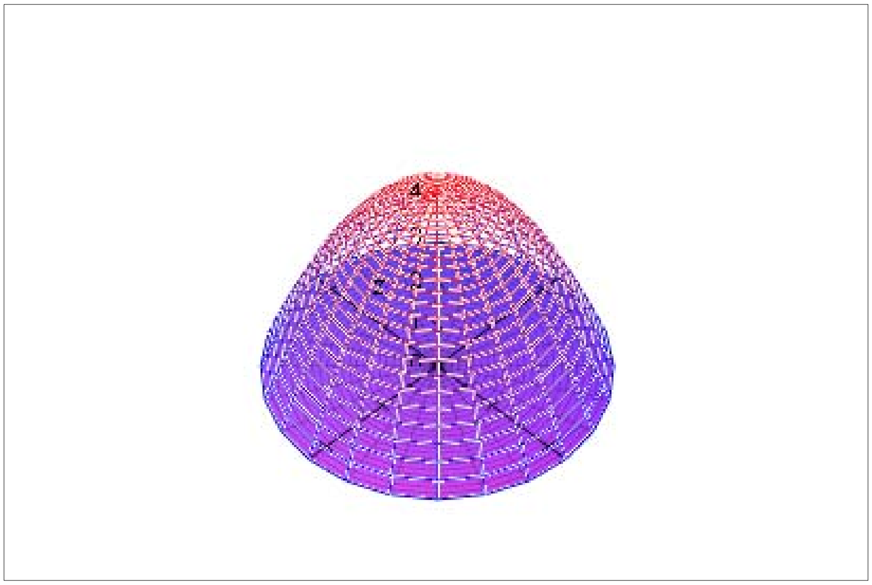
Evaluate the surface integral

$$\iint_S \vec{F} \cdot \vec{n} dS$$

where

$$\vec{F} = \left(xy\vec{i} - \frac{1}{2}y^2\vec{j} + z\vec{k}\right)$$

and the closed surface $S$ consists of the two surfaces $z = 4 - 3x^2 - 3y^2,\ 0 \le z \le 4$ on the top on the top with normal upward, and $z = 0$ on the bottom with normal downward.
Solution: $\left(r, \theta, 4 - 3r^2\right)$



We use the divergence theorem, namely

$$\iint_S \vec{F} \cdot \vec{n} dS = \iiint_E \nabla \cdot \vec{F} dV$$

where $E$ is the volume enclosed by $S$.

$$\nabla \cdot \vec{F} = y - y + 1 = 1$$

Note that $z = 0 \Rightarrow x^2 + y^2 = \frac{4}{3}$. Using cylindrical coordinates we have $0 \le z \le 4 - 3r^2$, $0 \le r \le \frac{2}{\sqrt{3}}$, and $0 \le \theta \le 2\pi$

$$\iiint_E \nabla \cdot \vec{F} dV = \int_0^{2\pi} \int_0^{\frac{2}{\sqrt{3}}} \int_0^{4-3r^2} (1) r dz dr d\theta = \int_0^{2\pi} \int_0^{\frac{2}{\sqrt{3}}} \left(4r - 3r^3\right) dr d\theta = \int_0^{2\pi} \left[2r^2 - \frac{3}{4} r^4\right]_0^{\frac{2}{\sqrt{3}}} d\theta$$

$$= \int_0^{2\pi} \left[2\left(\frac{4}{3}\right) - \left(\frac{3}{4}\right)\left(\frac{16}{9}\right)\right] d\theta = \int_0^{2\pi} \left(\frac{4}{3}\right) d\theta = \frac{8\pi}{3}$$

Alternatively, we will calculate the suface integral directly. Let $S_1$ denote the portion of the paraboloid on top and $S_2$ denote the disc on the bottom.

For $S_1$ we parametrize the surface using x and y as the parameters. Thus

$$\vec{r}(x,y) = \left\langle x, y, 4 - 3\left(x^2 + y^2\right)\right\rangle$$
$$\vec{r}_x(x,y) = \langle 1, 0, -6x \rangle$$
$$\vec{r}_y(x,y) = \langle 0, 1, -6y \rangle$$

The domain of $\vec{r}(x,y)$ is the disc $D = \left\{(x,y) | 0 \leqq x^2 + y^2 \leqq \frac{4}{3}\right\}$.

Then,

$$\vec{r}_x \times \vec{r}_y = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -6x \\ 0 & 1 & -6y \end{vmatrix} = 6x\vec{i} + 6y\vec{j} + \vec{k}$$

We observe that the z component is positive, so we have the correct orientaion of the normal.

$$
\begin{aligned}
\iint_{S_1} \vec{F} \cdot \vec{n} dS &= \iint_D \left\langle xy, -\frac{1}{2}y^2, 4 - 3\left(x^2 + y^2\right)\right\rangle \cdot \langle 6x, 6y, 1\rangle dA_{xy} \\
&= \iint_D \left[6x^2y - 3y^3 + 4 - 3\left(x^2 + y^2\right)\right] dA_{xy} \\
&= \int_0^{2\pi} \int_0^{\frac{2}{\sqrt{3}}} \left[6r^3 \cos^2\theta \sin\theta - 3r^3 \sin^3\theta + 4 - 3r^2\right] r dr d\theta \\
&= \int_0^{2\pi} \int_0^{\frac{2}{\sqrt{3}}} \left[6r^4 \cos^2\theta \sin\theta - 3r^4 \sin^3\theta + 4r - 3r^3\right] dr d\theta \\
& \int_0^{2\pi} \left[\frac{6}{5} r^5 \cos^2\theta \sin\theta - \frac{3}{4} r^5 \sin^3\theta + 2r^2 - \frac{3}{4} r^4\right] \Big|_{r=0}^{r=\frac{2}{\sqrt{3}}} d\theta \\
&= \int_0^{2\pi} \left[\frac{6}{5}\left(\frac{2}{\sqrt{3}}\right)^5 \cos^2\theta \sin\theta - \frac{3}{4}\left(\frac{2}{\sqrt{3}}\right)^5 \sin^3\theta + 2\frac{4}{3} - \frac{3}{4}\frac{16}{9}\right] d\theta \\
&= \frac{6}{5}\left(\frac{2}{\sqrt{3}}\right)^5 \left[\frac{-\cos^3\theta}{3}\right]_{\theta=0}^{\theta=2\pi} - \frac{3}{4}\left(\frac{2}{\sqrt{3}}\right)^5 \left[\frac{-\sin^2\theta \cos\theta - 2\cos\theta}{3}\right]_{\theta=0}^{\theta=2\pi} + \frac{4}{3} 2\pi \\
&= \frac{8\pi}{3}
\end{aligned}
$$

For $S_2$, we also parametrize using $x$ and $y$, but now $z = 0$ so it's much simpler.

$$
\begin{aligned}
\vec{r}(x,y) &= \langle x, y, 0\rangle \\
\vec{r}_x(x,y) &= \langle 1, 0, 0\rangle = \vec{i} \\
\vec{r}_y(x,y) &= \langle 0, 1, 0\rangle = \vec{j}
\end{aligned}
$$

The domain is the same disc,$D$, as for $S_1$.

$$\vec{r}_x \times \vec{r}_y = \vec{i} \times \vec{j} = \vec{k}$$

We observe that this vector points upward and we need the downward normal, so we use the negative and have

$$
\begin{aligned}
\iint_{S2} \vec{F} \cdot \vec{n} dS &= \iint_D \left\langle xy, -\frac{1}{2}y^2, 0\right\rangle \cdot \langle 0, 0, 1\rangle dA_{xy} \\
&= 0
\end{aligned}
$$

Finally

$$
\begin{aligned}
\iint_S \vec{F} \cdot \vec{n} dS &= \iint_{S_1} \vec{F} \cdot \vec{n} dS + \iint_{S2} \vec{F} \cdot \vec{n} dS \\
&= \frac{8\pi}{3} + 0
\end{aligned}
$$

b) (12 points)
Let

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Find $e^{At}$.
Solution:

$$e^{At} = \sum_{n=0}^{\infty} \frac{A^n t^n}{n!} = I + At + \frac{A^2 t^2}{2!} + \cdots + \frac{A^n t^n}{n!} + \cdots$$

Now

$$A^2 = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and

$$A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

so $A^n = 0$ for $n \geq 3$. Thus

$$e^{At} = I + At + \frac{A^2 t^2}{2!} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} t + \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{t^2}{2}$$

$$= \begin{bmatrix} 1 & t & 2t - \frac{t^2}{2} \\ 0 & 1 & -t \\ 0 & 0 & 1 \end{bmatrix}$$

# Problem 6

**a**) (**13 points**)

Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

Solution:

$$|A - rI| = \begin{vmatrix} 1-r & 2 \\ 3 & 2-r \end{vmatrix} = (1-r)(2-r) - 6 = r^2 - 3r - 4 = (r-4)(r+1)$$

so the eigenvalues are $r = -1, 4$. The system of equations for the eigenvectors is

$$(1-r)x_1 + 2x_2 = 0$$
$$3x_1 + (2-r)x_2 = 0$$

For $r = -1$ this becomes

$$2x_1 + 2x_2 = 0$$
$$3x_1 + 3x_2 = 0$$

so an eigenvector is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$. For $r = 4$ we have

$$-3x_1 + 2x_2 = 0$$
$$3x_1 - 2x_2 = 0$$

so an eigenvector is $\begin{bmatrix} 1 \\ \frac{3}{2} \end{bmatrix}$.

**b) (12 points)**
Solve the nonhomogeneous problem

$$x'(t) = Ax(t) + t\begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

where $A$ is the matrix above in 6 a).
Solution:

$$x_h = c_1e^{-t}\begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2e^{4t}\begin{bmatrix} 1 \\ \frac{3}{2} \end{bmatrix}$$

We assume that

$$x_p = t\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Then the system implies

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = t\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + t\begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

or

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = t\begin{bmatrix} a_1 + 2a_2 + 2 \\ 3a_1 + 2a_2 + 4 \end{bmatrix} + \begin{bmatrix} b_1 + 2b_2 \\ 3b_1 + 2b_2 \end{bmatrix}$$

This implies

$$a_1 + 2a_2 = -2$$
$$3a_1 + 2a_2 = -4$$

, Solution is: $\left[a_1 = -1, a_2 = -\frac{1}{2}\right]$. The equations for $b_1$ and $b_2$ are

$$b_1 + 2b_2 = -1$$
$$3b_1 + 2b_2 = -\frac{1}{2}$$

, Solution is: $\left[b_1 = \frac{1}{4}, b_2 = -\frac{5}{8}\right]$. Thus

$$x_p = t\begin{bmatrix} -1 \\ -\frac{1}{2} \end{bmatrix} + \begin{bmatrix} \frac{1}{4} \\ \frac{5}{8} \end{bmatrix}$$

and

$$x(t) = x_h(t) + x_p(t) = c_1 e^{-t}\begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 e^{4t}\begin{bmatrix} 1 \\ \frac{3}{2} \end{bmatrix} + t\begin{bmatrix} -1 \\ -\frac{1}{2} \end{bmatrix} + \begin{bmatrix} \frac{1}{4} \\ \frac{5}{8} \end{bmatrix}$$

SNB Check:

$$\frac{dx_1}{dt} = x_1 + 2x_2 + 2t$$
$$\frac{dx_2}{dt} = 3x_1 + 2x_2 + 4t$$

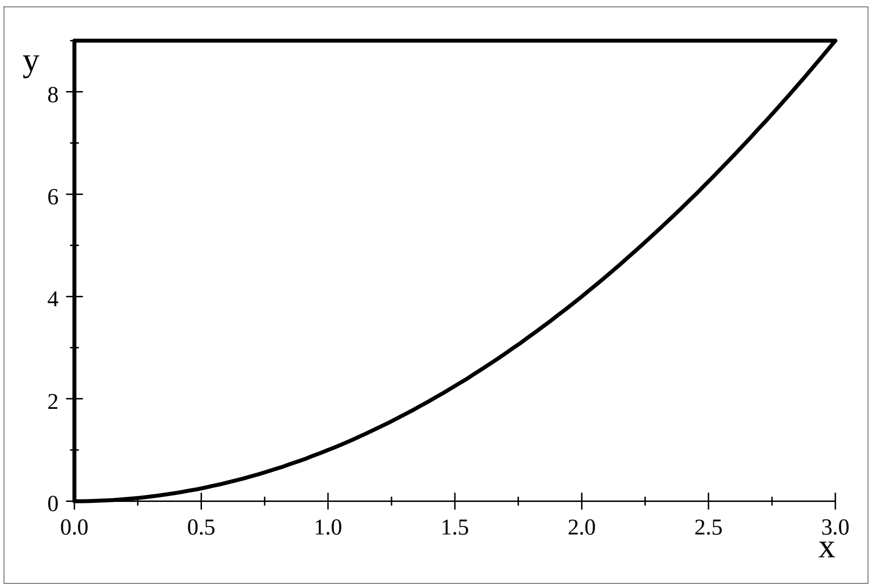
, Exact solution is: $\left[x_1(t) = \frac{2}{3}C_2 e^{4t} - C_1 e^{-t} - t + \frac{1}{4}, x_2(t) = C_1 e^{-t} - \frac{1}{2}t + C_2 e^{4t} - \frac{5}{8}\right]$

## Problem 7

**a**) (**13 points**)
Evaluate the integral

$$\int_0^2 \int_{x^2}^9 x^3 e^{y^3} dydx$$

Sketch the region of integration.
Solution: $x^2$




We change the order of integration in order to be able to evaluate the integral.

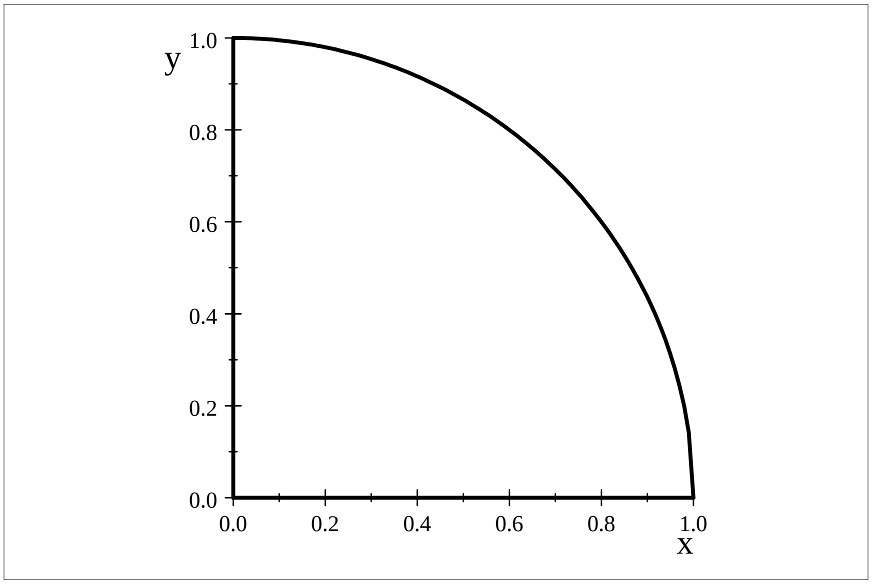
$$\int_0^2 \int_{x^2}^9 x^3 e^{y^3} dydx = \int_0^9 \int_0^{\sqrt{y}} e^{y^3} x^3 dxdy = \int_0^9 e^{y^3}\left[\frac{x^4}{4}\right]_0^{\sqrt{y}} dy = \frac{1}{4}\int_0^9 e^{y^3}\left(y^2\right)dy = \left.\frac{1}{4(3)}e^{y^3}\right]_0^9 = \frac{1}{12}\left[e^{9^3} - 1\right]$$

**b**) (**12 points**)
Evaluate

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \cos\left(x^2 + y^2\right) dxdy$$

Sketch the region of integration.
Solution: We have $0 \le x \le \sqrt{1-y^2}$ and $0 \le y \le 1$. Now $x = \sqrt{1-y^2}$ implies that $x^2 + y^2 = 1$,

which is the unit circle. Since $x$ is positive and $y$ is between 0 and 1, then the region of integration is the part of the unit circle in the first quadrant. Thus in polar coordinates $0 \leq r \leq 1$ and $0 \leq \theta \leq \frac{\pi}{2}$
$\sqrt{1-y^2}$




Switching to polar coordinates we have

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \cos\left(x^2+y^2\right) dxdy = \int_0^{\frac{\pi}{2}} \int_0^1 \cos\left(r^2\right) r dr d\theta = \int_0^{\frac{\pi}{2}} \left[ \frac{\sin\left(r^2\right)}{2} \right]_0^1 d\theta = \frac{\pi}{4} \sin(1)$$

## Problem 8

**a**) (**13 points**)

Consider the system

$$\begin{aligned} x_1 + 3x_2 - x_3 &= a \\ x_1 + 2x_2 &= b \\ 3x_1 + 7x_2 - x_3 &= c \end{aligned}$$

Find the conditions on $a, b$, and $c$ so that this system has solution. What happens when $a = b = 1$ and $c = 3$? What happens when $a = 1, b = 0$, and $c = -1$?

Solution: $\begin{bmatrix} 1 & 3 & -1 & a \\ 1 & 2 & 0 & b \\ 3 & 7 & -1 & c \end{bmatrix} \rightarrow^{-R_1+R_2, -3R_1+R_3} \begin{bmatrix} 1 & 3 & -1 & a \\ 0 & -1 & 1 & b-a \\ 0 & -2 & 2 & c-3a \end{bmatrix} \rightarrow -2R_2+R_3 \begin{bmatrix} 1 & 3 & -1 & a \\ 0 & -1 & 1 & b-a \\ 0 & 0 & 0 & c-3a-2(b-a \end{bmatrix}$

Thus for a solution we must have $c - 3a - 2(b-a) = c - a - 2b = 0$.

If $a = b = 1$ and $c = 3$, then the condition holds and there is a solution. If $a = 1, b = 0$, and $c = -1$, then the condition does not hold, and there is no solution.

**b**) (**12 points**)

Write the initial value problem

$$y''' + 2y'' - y' - 2y = 0, \ \ y(0) = 1, y'(0) = 0, y''(0) = -1$$

as an equivalent system in normal form with initial conditions.

Solution: Let $x_1 = y, x_2 = y', x_3 = y''$. Then since $y''' = -2y'' + y' + 2y$

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= x_3 \\ x_3' &= 2x_1 + x_2 - 2x_3 \end{aligned}$$

we have $x' = Ax,$ where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix}$$

and

$$x(0) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

# Table of Integrals

| |
|---|
| $\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$ |
| $\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$ |
| $\int \sin^3 x dx = -\frac{1}{3}\sin^2 x \cos x - \frac{2}{3}\cos x + C$ |
| $\int \cos^3 x dx = \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x + C$ |