Ma 227 **Final Exam** **12/20/06**

Name: ________________________________

Lecture Section: ________________________________

I pledge my honor that I have abided by the Stevens Honor System.

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You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Directions: Answer all questions. The point value of each problem is indicated. If you need more work space, continue the problem you are doing on the **other side of the page it is on**.

There is a table of integrals at the end of the exam.

Score on Problem #1 ________

#2 ________

#3 ________

#4 ________

#5 ________

#6 ________

#7 ________

#8 ________

Total ________

Problem 1

a) (**16 points**)

Compute the curl and divergence of the vector field

$$\vec{F}(x,y,z) = \sin y\vec{i} + x\cos y\vec{j} + (-\sin z + z)\vec{k}$$

b) (**9 points**)

Does there exist a function $\phi(x,y,z)$ such that $\nabla\phi = \vec{F}$ where $\vec{F}$ is the vector field in 1a above. Why or why not? If yes. then find $\phi(x,y,z)$

Problem 2

a) (**12 points**)

Give an expression in polar coordinates for

$$\iint_S z ds$$

where S is the part of the paraboloid $z = x^2 + y^2$ under the plane $z = 4$. Sketch S. Do *not* evaluate the expression.

b) (**13 points**)

Use Stokes' Theorem to evaluate $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = xy\vec{i} + yz\vec{j} + zx\vec{k}$ and C is the triangle with vertices $(1,0,0), (0,1,0),$ and $(0,0,1),$ oriented counter-clockwise when viewed from above.

Problem 3

a) (**12 points**)

Let R be the region that lies below the parabola $y = 4x - x^2$ above the x –axis and above the line $y = -3x + 6.$ Sketch $R.$ Give two integrals with different orders of integration for the area of $R.$ Do *not* evaluate either of these integrals.

b) (**13 points**)

Give an expression in cylindrical coordinates for the volume of the solid region D bounded above by the paraboloid $z = 1 - x^2 - y^2$ and below by the plane $z = 1 - y.$ Do *not* evaluate this expression.

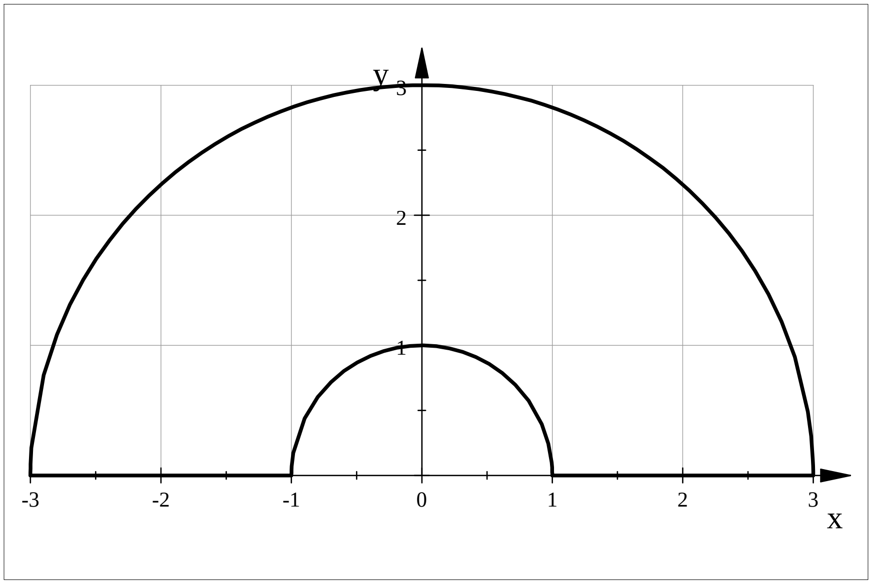
Problem 4

a) **(15 points)**

Evaluate the line integral

$$\oint_C \left(\arctan x + y^2\right)dx + \left(e^y - x^2\right)dy$$

where C is the path enclosing the annular region shown below.



b) (**10 points**)

Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 3 & 2 \\ -1 & 0 & 2 \\ 3 & 1 & -1 \end{bmatrix}.$$

Problem 5

a) **(13 points)**

Evaluate the surface integral

$$\iint_S \vec{F} \cdot \vec{n} ds = \iint_S \vec{F} \cdot d\vec{S}$$

where

$$\vec{F} = xy\vec{i} + \left(y^2 + e^{xz^2}\right)\vec{j} + \sin(xy)\vec{k}$$

and S is the surface of the region E bounded by the parabolic cylinder $z = 1 - x^2,$ and the planes $z = 0, y = 0,$ and $y + z = 2.$ Sketch $E.$

b) **(6 points)**

Let r be an eigenvalue for the constant square matrix A with corresponding eigenvector $u.$ Show that $x(t) = t^r u$ is a solution of

$$tx'(t) = Ax(t) \quad t > 0$$

5c) (**6 points**)

Show that if $t^r u$ is a solution of $tx'(t) = Ax(t) \quad t > 0,$ then r be an eigenvalue for the constant square matrix A with corresponding eigenvector u.

Problem 6

a) (**12 points**)

Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 6 \end{bmatrix}$$

b) (**13 points**)

Solve the nonhomogeneous system of equations

$$x'(t) = Ax(t) + \begin{bmatrix} -e^{t} \\ e^{-t} \end{bmatrix}$$

where A is the matrix above in 6a).

Problem 7

a) (**12 points**)
Evaluate the integral

$$\int_0^1 \int_x^1 \sin\left(y^2\right) dydx$$

Sketch the region of integration.

b) (**13 points**)
Evaluate

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dydx$$

Sketch the region of integration.

Problem 8

a) (**13 points**)

For what values of $c_1, c_2,$ and c_3 does the system

$$\begin{aligned} x_1 + 2x_2 - 3x_3 + x_4 &= c_1 \\ 3x_1 - x_2 + 2x_3 - 2x_4 &= c_2 \\ 5x_1 + 3x_2 - 4x_3 \qquad &= c_3 \end{aligned}$$

have a solution?

b) (**12 points**)

Express the initial value problem

$$y''' + ty'' + y' + 3y = e^t \sin t \qquad y(0) = 1, y'(0) = 2, y''(0) = 0$$

as a system of first order differential equation in normal form with an initial condition.

Table of Integrals

$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \sin^3 x dx = -\frac{1}{3}\sin^2 x \cos x - \frac{2}{3}\cos x + C$
$\int \cos^3 x dx = \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x + C$
$\int \sec^2\theta d\theta = \tan\theta + C$