Ma 227 **Final Exam** **5/15/06**

Name: ________________________________

Lecture Section: ________________________________

I pledge my honor that I have abided by the Stevens Honor System.

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You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Directions: Answer all questions. The point value of each problem is indicated. If you need more work space, continue the problem you are doing on the **other side of the page it is on**.

There is a table of integrals at the end of the exam.

Score on Problem #1 ________

#2 ________

#3 ________

#4 ________

#5 ________

#6 ________

#7 ________

#8 ________

Total ________

Problem 1

a) (**16 points**)

Compute the curl and divergence of the vector field

$$\vec{F}(x,y,z) = \left(y^2 + ze^{xz}\right)\vec{i} + 2xy\vec{j} + xe^{xz}\vec{k}$$

b) (**9 points**)

Does there exist a function $\phi(x,y,z)$ such that $\nabla\phi = \vec{F}$ where $\vec{F}$ is the vector field in 1a above. Why or why not? If yes. then find $\phi(x,y,z)$

Problem 2

a) (**12 points**)

Give an expression in polar coordinates for

$$\iint_S f(x,y,z)dS$$

where S is surface of the paraboloid $z = 4 - x^2 - y^2$ above the x,y –plane and $f(x,y,z) = x^2 + 2y^2 + z^2 - 4.$ Sketch $S.$ Do not evaluate the expression.

b) (13 points)

Use Stokes' Theorem to evaluate $\iint\limits_S \left(\nabla \times \vec{F}\right) \cdot \vec{n} dS$ where $\vec{F} = y\vec{i} + y\vec{j} + z\vec{k}$ and S is the surface defined by $S = \left\{ z = \left(x^2 + y^2\right)^{\frac{1}{3}},\ \ 0 \le z \le 2 \right\}$ Assume that S is oriented upwards.

Problem 3

a) **(12 points)**
Reverse the order of integration:

$$\int_{-1}^{2}\int_{y^2-3}^{y-1}(x^2+xy)\,dx\,dy$$

Be sure to sketch the region of integration. Do *not* evaluate.

b) **(13 points)**
Set up a triple integral in spherical coordinates to find the volume of the solid bounded above by the sphere $x^2+y^2+z^2=25$ and below by the plane $z=4$. Do *not* evaluate

Problem 4

a) (**15 points**)

Evaluate the line integral

$$\oint_C \left(xy + \ln\left(1 + x^2\right)\right) dx + x dy$$

where C is the triangle with vertices $(0,0),(1,0),(1,2)$ oriented in a counterclockwise manner. Sketch C.

b) (**10 points**)

Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}.$$

Problem 5

a) (**13 points**)

Evaluate the surface integral

$$\iint_S \vec{F} \cdot \vec{n} dS$$

where

$$\vec{F} = \left(2x^2y\vec{i} + 6y^2z\vec{j} + 2xz\vec{k}\right)$$

and S is the surface bounded by the unit cube $S = \{0 \le x,y,z \le 1\}$.

b) (**7 points**)

Let

$$A = \begin{bmatrix} 2 & 5 \\ 1 & -3 \end{bmatrix}$$

Find the characteristic polynomial $p(r)$ for A.

5c) (5 points)

Show that $p(A) = 0,$ where A is the matrix in 5b) and $p(r)$ is the characteristic polynomial of A.

Problem 6

a) (**12 points**)

Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

b) (**13 points**)
Solve the nonhomgeneous system of equations

$$x'(t) = Ax(t) + \begin{bmatrix} e^{2t} \\ 0 \end{bmatrix}$$

where A is the matrix above in 6a).

Problem 7

a) (**13 points**)
Evaluate the integral

$$\int_0^2 \int_{\frac{y}{2}}^1 \sqrt{6 - x^2}\, dxdy$$

Sketch the region of integration.

b) (**12 points**)
Convert to polar coordinates and evaluate

$$\int_0^1 \int_0^{\sqrt{3}\,x} 3xdydx$$

Sketch the region of integration.

Problem 8

a) (**13 points**)

Solve the system

$$\begin{aligned} x_1 + 2x_2 - 3x_3 + x_4 &= 0 \\ 3x_1 - x_2 + 2x_3 - 2x_4 &= 4 \\ 5x_1 + 3x_2 - 4x_3 \qquad &= 4 \end{aligned}$$

b) (**12 points**)

Give one differential equation that is equivalent to the system

$$x'(t) = \begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 3\sin(2t) \end{bmatrix}$$

Table of Integrals

$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$
$\int \sin^3 x dx = -\frac{1}{3}\sin^2 x \cos x - \frac{2}{3}\cos x + C$
$\int \cos^3 x dx = \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x + C$
$\int \sec^2 \theta d\theta = \tan\theta + C$