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Ma 227 **Final Exam** **12/17/07**

Name: ________________________________ **ID**: _________________________

Lecture Section: ________________________________

I pledge my honor that I have abided by the Stevens Honor System.

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You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Directions: Answer all questions. The point value of each problem is indicated. If you need more work space, continue the problem you are doing on the **other side of the page it is on**.

There is a table of integrals at the end of the exam.

Score on Problem #1 ________

#2 ________

#3 ________

#4 ________

#5 ________

#6 ________

#7 ________

#8 ________

Total ________

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Problem 1

a) (**13 points**)

Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

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b) **(12 points)**
Solve the nonhomogeneous problem

$$x'(t) = Ax(t) + \begin{bmatrix} e^{-t} \\ 2e^{-t} \end{bmatrix}$$

where A is the matrix above in 1a).

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Problem 2

a) (**12 points**)

Give an integral in cylindrical coordinates for the volume of the solid region bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 3$. Sketch the solid region. Do *not* evaluate this integral.

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b) (**13 points**)

Evaluate the surface integral $\iint_S \left(\nabla \times \vec{F}\right) \cdot \vec{n}\,dS = \iint_S \left(\nabla \times \vec{F}\right) \cdot d\vec{S},$ where

$$\vec{F}(x,y,z) = e^{xy}\cos z\vec{i} + x^2 z\vec{j} + xy\vec{k}$$

and S is the hemisphere $x = \sqrt{1 - y^2 - z^2}$ oriented in the direction of the positive x –axis.

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Problem 3

a) **(12 points)**

Evaluate

$$\int_0^1 \int_x^1 \cos\left(y^2\right) dy dx$$

b) **(13 points)**

Give two different integral expressions for the area of the region bounded by the parabolas $x = y^2$ and $x = 8 - y^2$. Do *not* evaluate these expressions. Be sure to sketch the area.

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Problem 4

a) (**10 points**)

Evaluate the line integral

$$\int_C zdx + xdy + ydz$$

where C is given by $x = t^2, y = t^3, z = t^2,\ \ 0 \leq t \leq 1.$

b) (**15 points**)

Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

and then use it solve the system $AX = \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}.$

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Problem 5

a) **(15 points)**

Verify the divergence theorem for the vector field $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$.

b) **(10 points)**

Let

$$D = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Find e^{Dt}.

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Problem 6

a) (**10 points**)

Find a function $\phi(x, y, z)$ such that

$$\nabla\phi = \vec{F}(x, y, z) = \sin y\vec{i} + x\cos y\vec{j} + (z - \sin z)\vec{k}$$

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6 b) (**15 points**)

Verify that Green's Theorem is true for the line integral

$$\int_C y\,dx + \left(x + y^2\right)dy$$

where C is the ellipse $4x^2 + 9y^2 = 36$ with counterclockwise orientation.

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Problem 7

a) (**13 points**)

Consider the two curves $x^2 + y^2 = 2y$ and $x^2 + y^2 = 2x$. Give an integral or integrals in polar coordinates for the area between the two curves. Be sure to sketch the area between the two curves. Do *not* evaluate the integral or integrals.

b) (**12 points**)

Evaluate

$$\iiint_E z\,dV$$

where E lies between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in the first octant.

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Problem 8

a) (**13 points**)

It can be shown that the Cauchy-Euler system

$$tx'(t) = Ax(t) \quad t > 0$$

where A is a constant matrix, has a nontrivial solutions of the form

$$x(t) = t^r u$$

if and only if r is an eigenvalue of A and u is a corresponding eigenvector. Use this information to solve the system

$$tx'(t) = \begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix} x(t) \quad t > 0$$

b) (**12 points**)

Rewrite the scalar equation

$$\frac{d^3y}{dt^3} - \frac{dy}{dt} + y = \cos t$$

as a first order system in normal form. Express the system in the matrix form $x' = Ax + f$.

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Table of Integrals

$$\int \sin^2 x\,dx = -\tfrac{1}{2}\cos x \sin x + \tfrac{1}{2}x + C$$
$$\int \cos^2 x\,dx = \tfrac{1}{2}\cos x \sin x + \tfrac{1}{2}x + C$$
$$\int \sin^3 x\,dx = -\tfrac{1}{3}\sin^2 x \cos x - \tfrac{2}{3}\cos x + C$$
$$\int \cos^3 x\,dx = \tfrac{1}{3}\cos^2 x \sin x + \tfrac{2}{3}\sin x + C$$
$$\int \left(\cos^2 x - \sin^2 x\right) dt = \tfrac{1}{2}\sin 2x + C$$