Name:________________________________ ID#:________________________

Ma 227 **Final Exam Solutions** **12/17/07**

Name: ________________________________

Lecture Section: ________________________________

I pledge my honor that I have abided by the Stevens Honor System.

____

You may not use a calculator, cell phone, or computer while taking this exam. All work must be shown to obtain full credit. Credit will not be given for work not reasonably supported. When you finish, be sure to sign the pledge.

Directions: Answer all questions. The point value of each problem is indicated. If you need more work space, continue the problem you are doing on the **other side of the page it is on**.

There is a table of integrals at the end of the exam.

Score on Problem #1 ________

#2 ________

#3 ________

#4 ________

#5 ________

#6 ________

#7 ________

#8 ________

Total ________

Name:_________________________________ ID#:_________________________

Problem 1

a) (**13 points**)

Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

Solution:

$$\begin{vmatrix} 2-r & 2 \\ 2 & 2-r \end{vmatrix} = (2-r)^2 - 4 = r^2 - 4r = r(r-4) = 0$$

Thus the eigenvalues are $r = 0, 4.$ The system $(A - rI)X = 0$ is

$$(2-r)x_1 + 2x_2 = 0$$
$$2x_1 + (2-r)x_2 = 0$$

For $r = 0$ we have

$$x_1 + x_2 = 0 \quad \text{or} \quad x_2 = -x_1$$

so the eigenvector corresponding to $r = 0$ is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

For $r = 4$ we have

$$-2x_1 + 2x_2 = 0 \quad \text{or} \quad x_1 = x_2$$

Thus the eigenvector for $r = 4$ is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

SNB check $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$, eigenvectors: $\left\{\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right\} \leftrightarrow 0, \left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\} \leftrightarrow 4$

b) (**12 points**)

Solve the nonhomogeneous problem

$$x'(t) = Ax(t) + \begin{bmatrix} e^{-t} \\ 2e^{-t} \end{bmatrix}$$

where A is the matrix above in 1a).

Solution:

$$x_h = c_1 e^{0t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_2 e^{4t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Let

$$x_p = \begin{bmatrix} ae^{-t} \\ be^{-t} \end{bmatrix}$$

Then the DE implies

Name:________________________________ ID#:________________________

$$\begin{bmatrix} -ae^{-t} \\ -be^{-t} \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} ae^{-t} \\ be^{-t} \end{bmatrix} + \begin{bmatrix} e^{-t} \\ 2e^{-t} \end{bmatrix}$$

or

$$\begin{bmatrix} -ae^{-t} \\ -be^{-t} \end{bmatrix} = \begin{bmatrix} e^{-t} + 2ae^{-t} + 2be^{-t} \\ 2e^{-t} + 2ae^{-t} + 2be^{-t} \end{bmatrix}$$

Therefore

$$-a = 2a + 2b + 1$$
$$-b = 2a + 2b + 2$$

or

$$3a + 2b = -1$$
$$2a + 3b = -2$$

, Solution is: $\left[a = \frac{1}{5}, b = -\frac{4}{5}\right]$

$$x_p = \begin{bmatrix} \frac{1}{5}e^{-t} \\ -\frac{4}{5}e^{-t} \end{bmatrix}$$

and

$$x(t) = x_h + x_p = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_2 e^{4t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{1}{5}e^{-t} \\ -\frac{4}{5}e^{-t} \end{bmatrix}$$

SNB check

$$x_1' = 2x_1 + 2x_2 + e^{-t}$$
$$x_2' = 2x_1 + 2x_2 + 2e^{-t}$$

, Exact solution is: $\left\{\left[x_1(t) = C_3 + \frac{1}{5}e^{-t} + C_4 e^{4t}, x_2(t) = C_4 e^{4t} - \frac{4}{5}e^{-t} - C_3\right]\right\}$

Problem 2

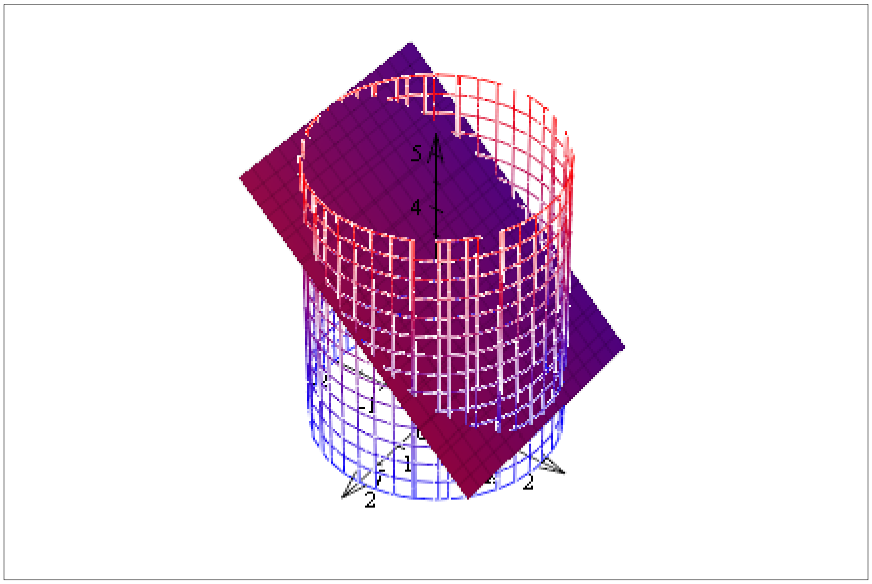
a) **(12 points)**

Give an integral in cylindrical coordinates for the volume of the solid region bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 3.$ Sketch the solid region. Do *not* evaluate this integral.

Solution:

$x^2 + y^2 = 4$

Name:________________________________ ID#:________________________



$$\text{Volume}=\iiint\limits_V r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^2\int_0^{3-r\sin\theta} r\,dz\,dr\,d\theta$$

b) (13 points)

Evaluate the surface integral $\iint\limits_S (\nabla\times\vec{F})\cdot\vec{n}dS = \iint\limits_S (\nabla\times\vec{F})\cdot d\vec{S}$, where

$$\vec{F}(x,y,z) = e^{xy}\cos z\vec{i} + x^2 z\vec{j} + xy\vec{k}$$

and S is the hemisphere $x = \sqrt{1-y^2-z^2}$ oriented in the direction of the positive x –axis.

Solution: We use Stokes theorem. Thus

$$\iint\limits_S (\nabla\times\vec{F})\cdot\vec{n}dS = \iint\limits_S (\nabla\times\vec{F})\cdot d\vec{S} = \oint\limits_C \vec{F}\cdot d\vec{r}$$

The boundary of S is the circle $y^2+z^2 = 1, x = 0.$ Therefore

$$C: \quad x = 0,\ \ y(t) = \cos t,\quad z(t) = \sin t\ \ 0 \le t \le 2\pi$$

so

$$\vec{r}(t) = 0\vec{i} + \cos t\vec{j} + \sin t\vec{k} \Rightarrow \vec{r}'(t) = -\sin t\vec{j} + \cos t\vec{k}$$

$$\vec{F}(0,\cos t,\sin t) = \cos(\sin t)\vec{i} + 0\vec{j} + 0\vec{k}$$

so

$$\oint\limits_C \vec{F}\cdot d\vec{r} = \int_0^{2\pi} 0dt = 0$$

Problem 3

a) (12 points)

Evaluate

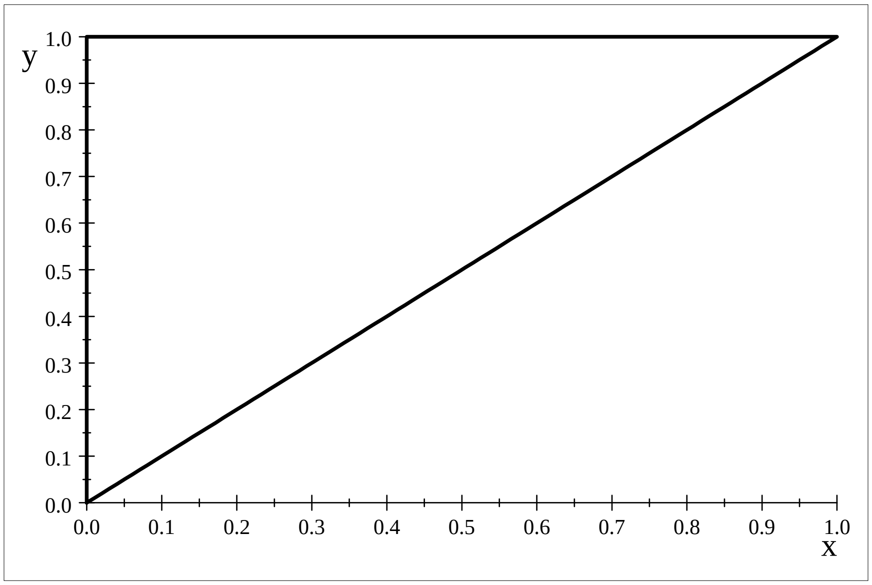
$$\int_0^1\int_x^1 \cos(y^2)\,dydx$$

Solution: We reverse the order of integration. Now y goes from $y = x$ to $y = 1$ and x goes from 0 to 1.

Name:_________________________________ ID#:_________________________

Therefore, the region of integration is

x



Therefore

$$\int_0^1 \int_x^1 \cos\left(y^2\right) dy dx = \int_0^1 \int_0^y \cos\left(y^2\right) dx dy$$

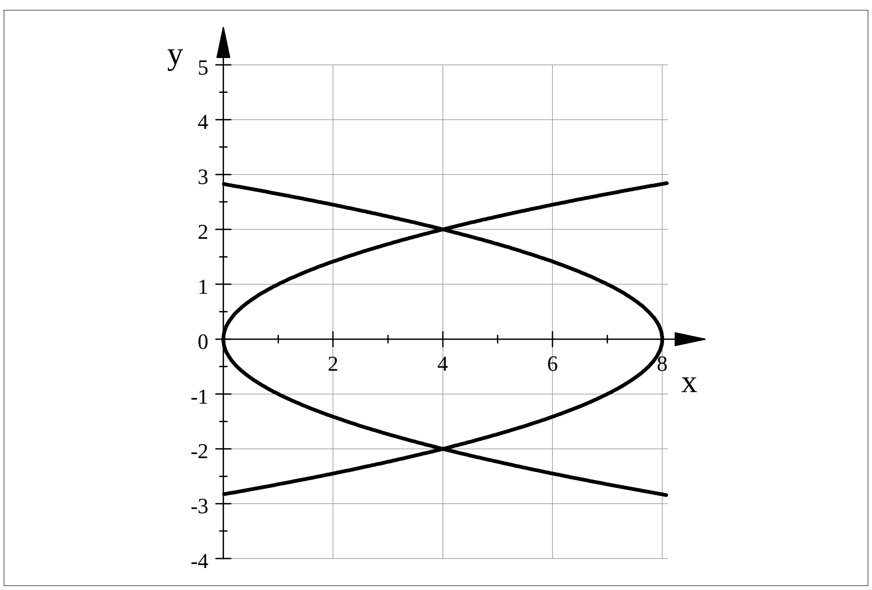
$$= \int_0^1 y \cos\left(y^2\right) dy = \left.\frac{\sin\left(y^2\right)}{2}\right]_0^1 = \frac{1}{2}\sin 1$$

b) (**13 points**)

Give two different integral expressions for the area of the region bounded by the parabolas $x = y^2$ and $x = 8 - y^2$. Do *not* evaluate these expressions. Be sure to sketch the area.

Solution:

$x = y^2$



Name:________________________________ ID#:________________________

The curves intersect when $y^2 = 8 - y^2$ that is, when $y = \pm 2$. Thus at the points $(4,2)$ and $(4,-2)$ Therefore

$$\begin{aligned}\text{Area} &= \iint_R dA \\ &= \int_{-2}^{2}\int_{y^2}^{8-y^2} dxdy \\ &= \int_0^4\int_{-\sqrt{x}}^{\sqrt{x}} dydx + \int_4^8\int_{-\sqrt{8-x}}^{\sqrt{8-x}} dydx\end{aligned}$$

Problem 4

a) (**10 points**)

Evaluate the line integral

$$\int_C zdx + xdy + ydz$$

where C is given by $x = t^2, y = t^3, z = t^2,\ 0 \le t \le 1$.

Solutions: Here

$$\vec{r}(t) = t^2\vec{i} + t^3\vec{j} + t^2\vec{k} \quad \Rightarrow \vec{r}'(t) = 2t\vec{i} + 3t^2\vec{j} + 2t\vec{k}$$

$$\vec{F}(x,y,z) = z\vec{i} + x\vec{j} + y\vec{k}$$

so

$$\vec{F}(t) = t^2\vec{i} + t^2\vec{j} + t^3\vec{k}$$

$$\begin{aligned}\int_C zdx + xdy + ydz &= \int_0^1 \left(t^2(2t) + t^2\left(3t^2\right) + t^3(2t)\right)dt \\ &= \int_0^1 \left(2t^3 + 5t^4\right)dt = \left.\frac{t^4}{2} + t^5\right]_0^1 = \frac{3}{2}\end{aligned}$$

b) (**15 points**)

Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

and then use it solve the system $AX = \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}$.

Solution:

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \to^{-R_1+R_3} \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \end{bmatrix} \to^{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 2 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Name:________________________________ ID#:________________________

$$\to^{-2R_2+R_3}\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 & -2 \end{bmatrix}\to^{-R_3+R_1}\begin{bmatrix} 1 & 0 & 0 & -1 & -1 & 2 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 & -2 \end{bmatrix}$$

Therefore

$$A^{-1} = \begin{bmatrix} -1 & -1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix}$$

SNB check: $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, inverse: $\begin{bmatrix} -1 & -1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix}$

We note that the solution of $AX = \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}$ is $X = A^{-1}\begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix} =$
$\begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$

Problem 5

a) (**15 points**)

Verify the divergence theorem for the vector field $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$.

Solution: We have to show that

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{N} ds = \iiint_V \operatorname{div} \vec{F} dv$$

We begin with the surface integral and use spherical coordinates to parametrize it. The $\rho = a$ and

$$\vec{r}(\varphi, \theta) = x\vec{i} + y\vec{j} + z\vec{k} = a\sin\varphi\cos\theta\vec{i} + a\sin\varphi\sin\theta\vec{j} + a\cos\varphi\vec{k} \quad 0 \le \varphi \le \pi, \quad 0 \le \theta \le 2\pi$$

Therefore

$$\vec{r}_\varphi = a\cos\varphi\cos\theta\vec{i} + a\cos\varphi\sin\theta\vec{j} - a\sin\varphi\vec{k}$$

and

$$\vec{r}_\theta = -a\sin\varphi\sin\theta\vec{i} + a\sin\varphi\cos\theta\vec{j} + 0\vec{k}$$

Name:_______________________________ ID#:________________________

$$\vec{N} = \vec{r}_\varphi \times \vec{r}_\theta = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a\cos\varphi\cos\theta & a\cos\varphi\sin\theta & -a\sin\varphi \\ -a\sin\varphi\sin\theta & a\sin\varphi\cos\theta & 0 \end{vmatrix}$$

$$= a^2\sin^2\varphi\cos\theta\,\vec{i} + a^2\sin^2\varphi\sin\theta\,\vec{j} + \left(a^2\cos\varphi\sin\varphi\cos^2\theta + a^2\cos\varphi\sin\varphi\sin^2\theta\right)\vec{k}$$

$$= a^2\sin^2\varphi\cos\theta\,\vec{i} + a^2\sin^2\varphi\sin\theta\,\vec{j} + a^2\cos\varphi\sin\varphi\,\vec{k}$$

Note that at $\left(\frac{\pi}{2},\frac{\pi}{2}\right) \Rightarrow \vec{N} = a^2\vec{j}$ which points outward, so we use this $\vec{N}$.

$$\vec{F}(\varphi,\theta) = a\sin\varphi\cos\theta\,\vec{i} + a\sin\varphi\sin\theta\,\vec{j} + a\cos\varphi\,\vec{k}$$

$$\iint_S \vec{F}\cdot d\vec{S} = \iint_S \vec{F}\cdot\vec{N}ds = \int_0^{2\pi}\int_0^{\pi}\left(a^3\sin^3\varphi\cos^2\theta + a^3\sin^3\varphi\sin^2\theta + a^3\cos^2\varphi\sin\varphi\right)d\varphi d\theta$$

$$= a^3\int_0^{2\pi}\int_0^{\pi}\left(\sin^3\varphi + \cos^2\varphi\sin\varphi\right)d\varphi d\theta = a^3\int_0^{2\pi}\left(\frac{1}{12}\cos 3\varphi - \frac{3}{4}\cos\varphi - \frac{\cos^3\varphi}{3}\right)_0^{\pi}d\theta$$

$$= a^3\int_0^{2\pi}\left(-\frac{1}{12} + \frac{3}{4} + \frac{1}{3} - \frac{1}{12} + \frac{3}{4} + \frac{1}{3}\right)d\theta = 2a^3\int_0^{2\pi}d\theta$$

$$= 4\pi a^3$$

Also

$$\operatorname{div}\vec{F} = 3$$

so

$$\iiint_V \operatorname{div}\vec{F}dv = \int_0^{2\pi}\int_0^{\pi}\int_0^{a}\left(3\rho^2\sin\varphi\right)d\rho d\varphi d\theta$$

$$= 3\frac{a^3}{3}\int_0^{2\pi}\int_0^{\pi}\sin\varphi d\varphi d\theta = a^3\int_0^{2\pi}(-\cos\varphi)_0^{\pi}d\theta$$

$$= 4\pi a^3$$

b) (10 points)
Let

$$D = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Find e^{Dt}.
Solution:

$$e^{Dt} = \begin{bmatrix} e^{-2t} & 0 & 0 \\ 0 & e^{4t} & 0 \\ 0 & 0 & e^{-t} \end{bmatrix}$$

Name:__________________________________ ID#:__________________________

SNB check: $e^{\begin{bmatrix} -2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \end{bmatrix} t} = \begin{bmatrix} e^{-2t} & 0 & 0 \\ 0 & e^{4t} & 0 \\ 0 & 0 & e^{-t} \end{bmatrix}$

Problem 6

a) (**10 points**)

Find a function $\phi(x,y,z)$ such that

$$\nabla\phi = \vec{F}(x,y,z) = \sin y\vec{i} + x\cos y\vec{j} + (z - \sin z)\vec{k}$$

Solution:

$$\phi_x = \sin y \quad \phi_y = x\cos y \quad \phi_z = z - \sin z$$

$$\phi_x = \sin y \quad \Rightarrow \phi = x\sin y + g(y,z)$$

Thus

$$\phi_y = x\cos y + \frac{\partial g}{\partial y} = x\cos y$$

Thus $\frac{\partial g}{\partial y} = 0$ and $g = h(z)$ so

$$\phi = x\sin y + h(z)$$

Then we have

$$\phi_z = h'(z) = z - \sin z$$

so $h(z) = \frac{z^2}{2} + \cos z + K$. Hence

$$\phi = x\sin y + \frac{z^2}{2} + \cos z + K$$

6 b) (**15 points**)

Verify that Green's Theorem is true for the line integral

$$\int_C ydx + \left(x + y^2\right)dy$$

where C is the ellipse $4x^2 + 9y^2 = 36$ with counterclockwise orientation.

Solution:

$$\oint_C P(x,y)dx + Q(x,y)dy = \iint_R (P_y - Q_x)dA$$

To calculate the line integral around the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 36$ we let $x(t) = 3\cos t$ and $y(t) = 2\sin t$, $0 \le t \le 2\pi$. Then $dx = -3\sin tdt, dy = 2\cos tdt$.

Name:________________________________ ID#:________________________

$$\int_C ydx + (x+y^2)dy = \int_0^{2\pi}\left((2\sin t)(-3\sin t) + \left(3\cos t + 4\sin^2 t\right)(2\cos t)\right)dt$$
$$= \int_0^{2\pi}\left(-6\sin^2 t + 6\cos^2 t + 8\sin^2 t\cos t\right)dt$$
$$= \int_0^{2\pi} 6\left(\cos^2 t - \sin^2 t + 8\sin^2 t\cos t\right)dt$$
$$= 3\sin 2t + \frac{8}{3}\sin^3 t\Big]_0^{2\pi} = 0$$

Also $P_y = 1 = Q_x$ so

$$\iint_R (P_y - Q_x)dA = \iint_R 0dA = 0$$

Problem 7

a) **(13 points)**

Consider the two curves $x^2 + y^2 = 2y$ and $x^2 + y^2 = 2x$. Give an integral or integrals in polar coordinates for the area between the two curves. Be sure to sketch the area between the two curves. Do *not* evaluate the integral or integrals.

Solution:

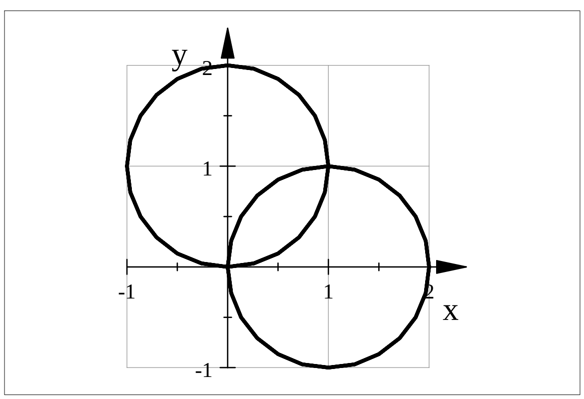
The two curves are given by

$$r = 2\sin\theta$$

and

$$r = 2\cos\theta$$

The graphs are given below.



$$A = \iint_R rdrd\theta$$

where R is the region common to both circles. The two circles intersect when

$$2\cos\theta = 2\sin\theta$$

or when

$$\tan\theta = 1$$

Name:________________________________ ID#:________________________

That is, at $\theta = \frac{\pi}{4}$. r goes from 0 to the circle $r = 2\sin\theta$, for $0 \le \theta \le \frac{\pi}{4}$ and from 0 to $r = 2\cos\theta$ for $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$. Thus we need two integrals to express the area.

$$A = \int_0^{\frac{\pi}{4}} \int_0^{2\sin\theta} r dr d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} r dr d\theta$$

b) (**12 points**)
Evaluate

$$\iiint_E z dV$$

where E lies between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in the first octant.
Solution: We use spherical coordinates. The equation of the unit sphere is $\rho = 1$ whereas the equation of the other sphere is $\rho = 2$. Since E is in the first octant, then $0 \le \phi \le \frac{\pi}{2}$ and $0 \le \theta \le \frac{\pi}{2}$. Thus

$$\iiint_E z dV = \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_1^2 (\rho\cos\phi)\rho^2 \sin\phi d\rho d\theta d\phi$$

$$= \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \left.\frac{\rho^4}{4}\right]_1^2 \cos\phi\sin\phi d\theta d\phi$$

$$= \frac{16-1}{4}\left(\frac{\pi}{2}\right) \int_0^{\frac{\pi}{2}} \cos\phi\sin\phi d\theta d\phi$$

$$= \frac{15\pi}{8} \left.\frac{\sin^2\phi}{2}\right]_0^{\frac{\pi}{2}} = \frac{15\pi}{16}$$

Problem 8

a) (**13 points**)
It can be shown that the Cauchy-Euler system

$$tx'(t) = Ax(t) \quad t > 0$$

where A is a constant matrix, has a nontrivial solutions of the form

$$x(t) = t^r u$$

if and only if r is an eigenvalue of A and u is a corresponding eigenvector. Use this information to solve the system

$$tx'(t) = \begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix} x(t) \quad t > 0$$

Solution: We first find the eigenvectors of $\begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix}$.

$$\left|\begin{bmatrix} -4-r & 2 \\ 2 & -1-r \end{bmatrix}\right| = (4+r)(1+r) - 4 = r^2 + 5r$$

so the eigenvalues are $r = 0, -5$. The system $(A - rI)x = 0$ is

$$(-4-r)x_1 + 2x_2 = 0$$
$$2x_1 + (-1-r)x_2 = 0$$

Name:__________________________________ ID#:__________________________

$r = 0$ yields $-4x_1 + 2x_2 = 0$ or $2x_1 = x_2$. This gives the eigenvector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$. $r = -5$ leads to the equation $x_1 = -2x_2$ and hence the eigenvector $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$. Thus

$$x(t) = c_1 t^0 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 t^{-5} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

b) (**12 points**)
Rewrite the scalar equation

$$\frac{d^3y}{dt^3} - \frac{dy}{dt} + y = \cos t$$

as a first order system in normal form. Express the system in the matrix form $x' = Ax + f$.
Solution: Let

$$x_1(t) = y(t),\ \ x_2(t) = y'(t),\ x_3(t) = y''(t)$$

Then

$$\begin{aligned} x_1'(t) &= y'(t) = x_2(t) \\ x_2'(t) &= y''(t) = x_3(t) \\ x_3'(t) &= y'''(t) = y' - y + \cos t = x_2 - x_1 + \cos t \end{aligned}$$

Thus

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \cos t \end{bmatrix}$$

Name:________________________________ ID#:________________________

Table of Integrals

$$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$$

$$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$$

$$\int \sin^3 x dx = \frac{1}{12}\cos 3x - \frac{3}{4}\cos x - \frac{2}{3}\cos x + C$$

$$\int \cos^3 x dx = \frac{3}{4}\sin x + \frac{1}{12}\sin 3x + C :$$

$$\int \left(\cos^2 x - \sin^2 x\right) dt = \frac{1}{2}\sin 2x + C$$