Ma 227 **Final Exam** **12/22/09**

Name: ________________________________

Lecture Section: ____ (**A**: **Prof**. **Levine**, **B**: **Prof**. **Brady**)

You may not use a calculator, **cell phone**, **or computer while taking this exam**. **All work must be shown to obtain full credit**. **Credit will not be given for work not reasonably supported**. **When you finish**, **be sure to sign the pledge**.

Directions: Answer all questions. The point value of each problem is indicated. If you need more work space, continue the problem you are doing on the **other side of the page it is on**.

There is a table of integrals at the end of the exam.

Score on Problem #1 ________

#2 ________

#3 ________

#4 ________

#5 ________

#6 ________

#7 ________

#8 ________

Total ________

I pledge my honor that I have abided by the Stevens Honor System.

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Name:________________________________

Problem 1

a) (**13 points**)

Does the following system of equations have a unique solution or an infinite set of solutions or no solution? Find all solutions, if they exist.

$$\begin{aligned} x_1 + x_2 + 2x_3 + 2x_4 &= 6 \\ x_1 + 2x_2 + 3x_3 - x_4 &= 5 \\ 2x_1 + 3x_2 + 5x_3 + x_4 &= 11 \\ x_1 + 3x_2 + 4x_3 - 2x_4 &= 6 \end{aligned}$$

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b) (**12 points**)
Show that

$$\operatorname{div}(\operatorname{curl}(\vec{F})) = \nabla \cdot \left(\nabla \times \vec{F}\right) = \vec{0}.$$

(Assume that the mixed second partial derivatives are equal.) $\vec{F}(x,y,z) = P\vec{i} + Q\vec{j} + R\vec{k}.$

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Problem 2

a) (**10 points**)

Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}.$$

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b) (**15 points**)

Find the [real] general solution to

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 12e^{2t} \end{bmatrix}.$$

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Problem 3

a) (**12 points**)

Let R be the region in the first octant bounded by the plane $x + 2y + 3z = 6$. Sketch the region R. Set up three iterated integrals for the volume with the orders of integration as specified below.

$$\iiint dz\,dy\,dx \quad , \quad \iiint dy\,dx\,dz \quad , \quad \iiint dx\,dz\,dy$$

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b) (**13 points**)

The integral below represents the volume of a solid. Describe the solid. Evaluate the integral.

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} dzdydx$$

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Problem 4

a) (**10 points**)
Evaluate

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n}\, dS$$

where

$$\vec{F} = x^2 z^3 \,\vec{i} + 2xyz^3 \,\vec{j} + xz^4 \,\vec{k}$$

and S is the surface of the box with vertices $(\pm 1, \pm 2, \pm 3)$.

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b) (**15 points**)
Evaluate

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n}\, dS$$

where

$$\vec{F} = x^2\vec{i} + xy\vec{j} + z\vec{k}$$

and S is the portion of the paraboloid $z = x^2 + y^2$ below $z = 1$, oriented with the normal pointing downward.

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Problem 5

(25 points)

Verify Stokes' Theorem for the vector field $\vec{F} = -y\vec{i} + 2x\vec{j} + (z + x)\vec{k}$ over the upper hemisphere $x^2 + y^2 + z^2 = 1,\ z \geq 0,$ oriented by the outward pointing normal $\vec{n}$.

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Problem 6

a) (**13 points**)

It can be shown the for the force field

$$\vec{F}(x,y,z) = e^x \cos y\vec{i} - e^x \sin y\vec{j} + 2\vec{k}$$

that

$$\text{curl}\,\vec{F} = 0$$

Find the work done by $\vec{F}$ on an object moving along a curve C from $\left(0, \frac{\pi}{2}, 1\right)$ to $(1, \pi, 3)$.

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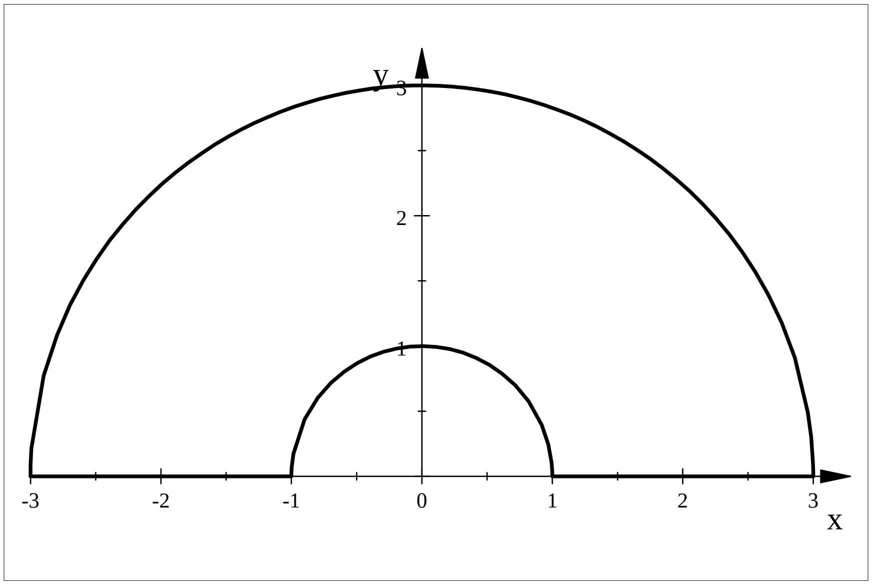
6 b) (**12 points**)
Evaluate

$$\oint_C \left(\arctan x + y^2\right)dx + \left(e^y - x^2\right)dy$$

where C is the path enclosing the annular region shown below, positively oriented.



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Problem 7

a) (**13 points**)
Calculate

$$\iint_R \left(x^2 + y^2\right)^{-2} dA$$

where R is the part of the circle centered at $(1, 0)$ of radius 1 to the right of the line $x = 1$ in the first quadrant. Be sure to sketch R.

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b) (**12 points**)
Evaluate

$$\iiint_E z\,dV$$

where E is the region within the cylinder $x^2 + y^2 = 4$, where $0 \le z \le y$.

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Problem 8

a) (**13 points**)

It can be shown that the Cauchy-Euler system

$$tx'(t) = Ax(t) \quad t > 0$$

where A is a constant matrix, has a nontrivial solutions of the form

$$x(t) = t^r u$$

if and only if r is an eigenvalue of A and u is a corresponding eigenvector. Use this information to solve the system

$$tx'(t) = \begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix} x(t) \quad t > 0$$

b) (**12 points**)

Rewrite the system of equations

$$x' = Ax + f$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} \text{ and } f = \begin{bmatrix} 0 \\ 0 \\ \cos t \end{bmatrix}$$

as a single scalar equation.

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Table of Integrals

$$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$$
$$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$$
$$\int \sin^3 x dx = -\frac{1}{3}\sin^2 x \cos x - \frac{2}{3}\cos x + C$$
$$\int \cos^3 x dx = \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x + C$$
$$\int \left(\cos^2 x - \sin^2 x\right) dt = \frac{1}{2}\sin 2x + C$$