**Ma 227** **Final Exam** **12/16/10**

**Name**: ________________________________

**Lecture Section**: ____ (**A**: **Prof**. **Levine**, **B**: **Prof**. **Brady**, **C**. **Prof**. **Strigul**)

**You may not use a calculator**, **cell phone**, **or computer while taking this exam**. **All work must be shown to obtain full credit**. **Credit will not be given for work not reasonably supported**. **When you finish**, **be sure to sign the pledge**.

**Directions**: Answer all questions. The point value of each problem is indicated. If you need more work space, continue the problem you are doing on the **other side of the page it is on**.

**There is a table of integrals at the end of the exam**.

Score on Problem #1 ________

#2a ________

#2b ________

#3 ________

#4 ________

#5 ________

#6 ________

#7 ________

#8 ________

Total ________

I pledge my honor that I have abided by the Stevens Honor System.

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## Problem 1

**a**)  (**13 points**)

Find the value of $c$ that makes it possible to solve the following system of equations and solve the system.  Is your solution unique or an infinite set?

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 2 \\ 2x_1 + 3x_2 - x_3 &= 5 \\ 3x_1 + 4x_2 + x_3 &= c \end{aligned}$$

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**b**) (**12 points**)

Let $\vec{F}(x,y,z)$ and $\vec{G}(x,y,z)$ be vector fields with continuous partial derivatives. Show that

$$div\left(\vec{F}\times\vec{G}\right) = \vec{G}\cdot curl\vec{F} - \vec{F}\cdot curl\vec{G}$$

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## Problem 2

**a**) (**10 points**)

Find the eigenvalues and eigenvectors of the matrix A.

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

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**b**) (**15 points**)

The eigenvalues of the matrix $\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$ are $2 + i$ and $2 - i$ and the corresponding eigenvectors are $\begin{bmatrix} -1 - i \\ 2 \end{bmatrix}$ and $\begin{bmatrix} -1 + i \\ 2 \end{bmatrix}$.

Find a [real] general solution to

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 25t \\ 0 \end{bmatrix}.$$

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## Problem 3

**a**)  (**25 points**)

Consider the triple integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \left(x^2 + y^2 + z^2\right) dzdydx.$$

i. Describe and sketch the region of integration.

ii. Give an equivalent triple integral in rectangular coordinates in a different order of integration.

iii. Give an equivalent triple integral in cylindrical coordinates.

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$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \left(x^2+y^2+z^2\right) dzdydx.$$

iv. Give an equivalent triple integral in spherical coordinates.

v. Use any of your equivalent triple integrals to evaluate the integral.

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# Problem 4

**a**) (**10 points**)

Evaluate $\int_C (2x - y)dx + (-x - 2y)dy$ where $C$ is given by $x = \cos\theta,\ y = \sin^2\theta,\ 0 \le \theta \le \frac{\pi}{2}$.

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**b)** (**15 points**)

Evaluate the surface integral $\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n}\, dS$ for $\vec{F} = z\,\vec{i} + x\,\vec{j} + y\,\vec{k}$ with $S$ the triangle with vertices $(1,0,0), (0,1,0)$ and $(0,0,1)$ oriented upward.

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## Problem 5

**(25 points)**

Verify the Divergence Theorem for the vector field $\vec{F} = y\vec{i} + yz\vec{j} + z^2\vec{k}$ where $S$ is the surface of the solid $E$ bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 5$.

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## Problem 6

**(25 points)**

Verify Green's Theorem for the line integral

$$\oint_C \left(xy^2 dx + x dy\right)$$

where $C$ is the unit circle centered at the origin oriented counterclockwise.

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## Problem 7

**a**) (**13 points**)
Calculate

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dydx$$

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**b)** (**12 points**)

Find the volume of solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$. Sketch the solid.

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## Problem 8

**a**) (**13 points**)

Let

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

Find $e^{At}$.

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**b**) (**12 points**)
Rewrite the equation

$$\frac{d^3y}{dt^3} - \frac{dy}{dt} + y = \sin t,\ y(0) = 1, y'(0) = 0, y''(0) = -4$$

as a system of differential equations in normal form with appropriate initial condition.

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# Table of Integrals

$$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$$
$$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$$
$$\int \sin^3 x dx = -\frac{1}{3}\sin^2 x \cos x - \frac{2}{3}\cos x + C$$
$$\int \cos^3 x dx = \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x + C$$
$$\int \left(\cos^2 x - \sin^2 x\right) dt = \frac{1}{2}\sin 2x + C$$