Ma 227 **Final Exam** **12/13/11**

Name: ________________________________

Lecture Section: ____ (**A and B**: **Prof**. **Levine**, **C**: **Prof**. **Brady**)

You may not use a calculator, **cell phone**, **or computer while taking this exam**. **All work must be shown to obtain full credit**. **Credit will not be given for work not reasonably supported**. **When you finish**, **be sure to sign the pledge**.

Directions: Answer all questions. The point value of each problem is indicated. If you need more work space, continue the problem you are doing on the **other side of the page it is on**.

There is a table of integrals at the end of the exam.

Score on Problem #1 ________

#2 ________

#3 ________

#4 ________

#5 ________

#6 ________

#7 ________

#8 ________

Total ________

I pledge my honor that I have abided by the Stevens Honor System.

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Name:________________________________

Problem 1

a) (**10 points**)

Find the eigenvalues and eigenvectors of the matrix A.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & -5 \\ 2 & 1 & -1 \end{bmatrix}$$

Name:________________________________

b) (**15 points**)

The eigenvalues of the matrix $\begin{bmatrix} 3 & 4 \\ -2 & -1 \end{bmatrix}$ are $1 + 2i$ and $1 - 2i$ and the corresponding eigenvectors are $\begin{bmatrix} -1 - i \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 + i \\ 1 \end{bmatrix}$.

Find a [real] general solution to

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 25t \\ 0 \end{bmatrix}.$$

Name:________________________________

Problem 2

a) **(10 points)**

Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ for $\vec{F}(x, y, z) = x\vec{i} + y\vec{j} + z^2\vec{k}$ and C one turn around the spiral $\vec{r} = \cos t\vec{i} + \sin t\vec{j} + t\vec{k}$ from $(1, 0, 0)$ to $(1, 0, 2\pi)$.

Name:________________________________

b) (**15 points**)

Consider the triple integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{1+\sqrt{1-x^2-y^2}} \left(x^2+y^2+z^2\right) dz\,dy\,dx.$$

i. Describe and sketch the region of integration.

ii. Give an equivalent triple integral in cylindrical coordinates.

iii. Give an equivalent triple integral in spherical coordinates.

Name:__________________________________

Problem 3

(25 points)

Evaluate the surface integral

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n}\, dS$$

where S is the surface of the portion of the cone $z^2 = x^2 + y^2$ in the first octant and below the plane $z = 4$ with downward (outward) normal and

$$\vec{F} = yz\vec{i} + xz\vec{j} + xy\vec{k}.$$

Name:________________________________

Problem 4

a) (**10 points**)

Show that, if all partial derivatives of $f(x,y,z)$ are continuous,

$$\text{curl}\big(\text{grad } f(x,y,z)\big) = \vec{0}.$$

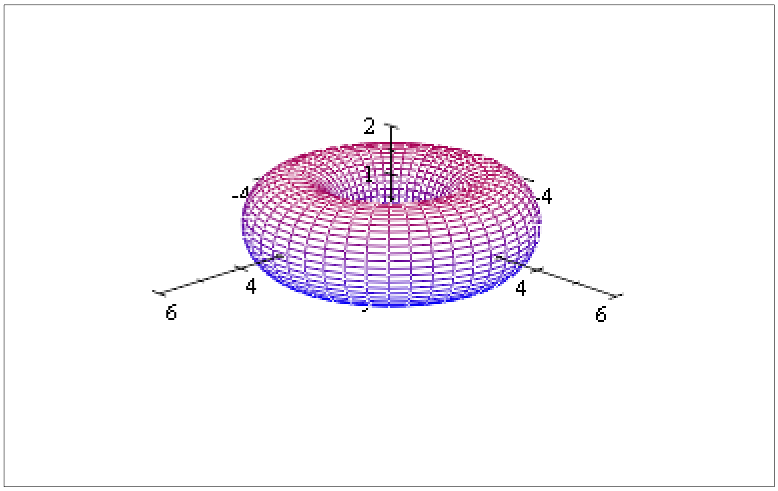
Name:________________________________

b) **(15 points)**

The figure below shows the torus obtained by rotating about the *z*-axis the circle in the *xz*-plane with center $(2,0,0)$ and radius 1.
Parametric equations for the torus are

$$
\begin{aligned}
x &= 2\cos\theta + \cos\alpha\cos\theta \\
y &= 2\sin\theta + \cos\alpha\sin\theta \\
z &= \sin\alpha \\
0 &\le \alpha \le 2\pi, \quad 0 \le \theta \le 2\pi.
\end{aligned}
$$

θ is the usual polar angle around the *z* axis and α is the angle around the circle in the *xz*-plane. Find the surface area of the torus.



Name:________________________________

Name:________________________________

Problem 5

a) (**13 points**)

Use Stokes' Theorem to evaluate

$$\iint_S \operatorname{curl}\vec{F} \cdot d\vec{S}$$

where $\vec{F} = z^2\vec{i} - 3xy\vec{j} + x^3y^3\vec{k}$ and S is the part of $z = 5 - x^2 - y^2$ above the plane $z = 1$. Assume S is oriented upwards.

Name:________________________________

b) (**12 points**)

Use the divergence theorem to evaluate

$$\iint_S \vec{F} \cdot d\vec{S}$$

where $\vec{F} = xy\vec{i} - \frac{1}{2}y^2\vec{j} + z\vec{k}$ and the surface S consists of the three surfaces, $z = 4 - 3x^2 - 3y^2$, $1 \le z \le 4,$ on the top, $x^2 + y^2 = 1,\ 0 \le z \le 1$ on the sides, and $z = 0$ on the bottom.

Name:________________________________

Problem 6 (25 points)

Verify Green's Theorem for the line integral

$$\oint_C \left(y^2 dx + 3xy dy \right)$$

where C is the upper half of the unit circle centered at the origin oriented counterclockwise.

Name:________________________________

Problem 7

a) (**13 points**)
The integral

$$\int_{-2}^{1}\int_{y^2}^{2-y} dxdy$$

gives the area of a region R in the x,y –plane. Sketch R and then give another expression for the area of R with the order of integration reversed. Do *not* evaluate this expression.

Name:________________________________

b) (12 points)
Find the volume of the region that lies under the sphere $x^2 + y^2 + z^2 = 9,$ above the plane $z = 0$ and inside the cylinder $x^2 + y^2 = 5.$ Sketch the solid.

Name:________________________________

Problem 8

a) (**8 points**)

Show that the characteristic polynomial for the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ -2 & -2 & -1 \end{bmatrix}$$

is

$$p(r) = -(r-1)^3$$

b) (**7 points**)

Show that

$$p(A) = -(A-I)^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where A is the matrix in part a)

Name:________________________________

c) (**10 points**)
Calculate e^{At}.

Name:________________________________

Table of Integrals

$$\int \sin^2 x dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$$
$$\int \cos^2 x dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$$
$$\int \sin^3 x dx = -\frac{1}{3}\sin^2 x \cos x - \frac{2}{3}\cos x + C$$
$$\int \cos^3 x dx = \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x + C$$
$$\int \left(\cos^2 x - \sin^2 x\right) dt = \frac{1}{2}\sin 2x + C$$