Ma 227 **Final Exam** **12/17/13**

Name: ________________________________

Lecture Section: ____ (**A** [**10 AM**] **and B** [**11 AM**]: **Prof**. **Levine**, **C** [**12PM**]: **Prof**. **Brady**)

You may not use a calculator, **cell phone**, **or computer while taking this exam**. **All work must be shown to obtain full credit**. **Credit will not be given for work not reasonably supported**. **When you finish**, **be sure to sign the pledge**.

Directions: Answer all questions. The point value of each problem is indicated. If you need more work space, continue the problem you are doing on the **other side of the page it is on**.

There is a table of integrals at the end of the exam.

Score on Problem #1a ________

#1b ________

#2a ________

#2b ________

#3a ________

#3b ________

#4a ________

#4b ________

#4c ________

#4d ________

#4e ________

#5a ________

#5b ________

#6 ________

#7a ________

#7b ________

#8a ________

#8b ________

Total ________

I pledge my honor that I have abided by the Stevens Honor System.

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Name:________________________________

Problem 1

Let

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 1 \\ -2 & 1 & -1 \end{bmatrix}.$$

a) (**13 points**)
Find all eigenvalues and eigenvectors of the matrix A.

Name:________________________________

b) (**12 points**)
Find a general solution for the system

$$\mathbf{x}'(t) = Ax(t), \mathbf{x}(0) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

where A is the matrix above.

Name:________________________________

Problem 2

a) (**12 points**)

Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}.$$

Name:________________________________

b) (**13 points**)
Let

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 3 & 4 \\ 1 & -2 & -1 \end{bmatrix}.$$

Eigenvectors and corresponding eigenvalues of A are

$$\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} \right\} \leftrightarrow 2, \left\{ \begin{bmatrix} 0 \\ -1+i \\ 1 \end{bmatrix} \right\} \leftrightarrow 1-2i, \left\{ \begin{bmatrix} 0 \\ -1-i \\ 1 \end{bmatrix} \right\} \leftrightarrow 1+2i.$$

Find all real solutions to the system of differential equations

$$\mathbf{x}' = A\mathbf{x} + \begin{bmatrix} 4e^t \\ -6e^t \\ 8e^t \end{bmatrix}.$$

Name:________________________________

Problem 3

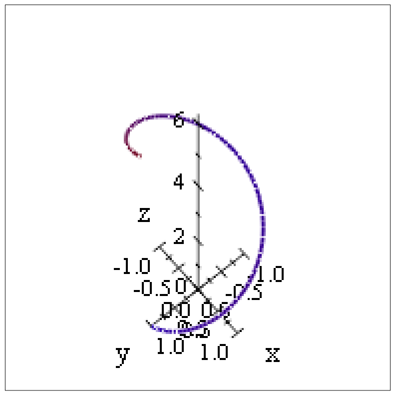
a) (**12 points**)
Evaluate the line integral

$$\int_C yze^{xyz}dx + xze^{xyz}dy + (xye^{xyz} + 1)dz$$

where C is one turn around a spiral defined by

$$x = \cos t, \quad y = \sin t, \quad z = t \quad 0 \le t \le 2\pi.$$

Here's a picture of C.



Name:________________________________

b) (**13 points**)
Find the area of the surface of the part of the right circular cone $z^2 = x^2 + y^2$ which is between the planes $z = 0$ and $z = 1$.

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Problem 4

(**25 points**)

Consider the triple integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} \left(x^2 + y^2 + z^2\right) dz\,dy\,dx.$$

a) Describe (in words and/or equations) and sketch the region of integration.

b). Give an equivalent triple integral in rectangular coordinates in a different order of integration.

c). Give an equivalent triple integral in cylindrical coordinates.

Name:_________________________________

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} \left(x^2+y^2+z^2\right) dzdydx.$$

d). Give an equivalent triple integral in spherical coordinates.

e). Use any of your equivalent triple integrals to evaluate the integral.

Name:________________________________

Problem 5

a) (**15 points**)

Show that

$$\oint_C \vec{F} \cdot d\vec{r} = 0$$

where

$$\vec{F} = \sin\left(x^2\right)\vec{i} + \left(e^{y^2} + x^2\right)\vec{j} + \left(z^4 + 2x^2\right)\vec{k}$$

and C consists of the line segment joining $(3,0,0)$ to $(0,2,0)$ followed by the line segment joining $(0,2,0)$ to $(0,0,1)$ followed by the line segment joining $(0,0,1)$ to $(3,0,0)$. Sketch C. Note: The plane through the points $(3,0,0)$, $(0,2,0)$, and $(0,0,1)$ has the equation

$$\frac{x}{3} + \frac{y}{2} + z = 1$$

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5b) (**10 points**)
Evaluate

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \vec{n}\, dS$$

where

$$\vec{F} = \left(x^2 + xz\right)\vec{i} + 2y^2\vec{j} - \frac{z^2}{2}\vec{k}$$

and S is surface of the cube bounded by $x = 0, x = 1. y = 0, y = 1, z = 0, z = 1.$

Name:________________________________

Problem 6

(25 points)
Verify Green's Theorem for the line integral

$$\oint_C \left(4x^3 + 2y^2\right)dx + (4xy + e^y)dy$$

where C is the boundary of the region between $y = x^2$ and $y = \sqrt{x}$. Sketch C.

Name:________________________________

Problem 7

a) (**13 points**)

Give two double integral expressions for the area of the region R bounded by $y = x - 6$ and $y^2 = x$. Be sure to sketch R. Do *not* evaluate these expressions.

Name:________________________________

7b) (**12 points**)

Give the expression in *cylindrical* coordinates for the volume of the solid inside both the cylinder $x^2 + y^2 = 4$ and the ellipsoid $4x^2 + 4y^2 + z^2 = 64$. Sketch the part of the volume in the first octant. Do *not* evaluate this expression.

Name:________________________________

Problem 8

a) (**13 points**)

Let

$$A = \begin{bmatrix} -2 & -1 & 1 \\ 2 & 1 & 0 \\ 3 & 1 & -3 \end{bmatrix}.$$

Find A^{-1}.

Name:________________________________

b) (**12 points**)
Rewrite the initial value problem for the system

$$x'(t) = Ax(t) + f(t) \qquad x(0) = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -e^t & t^2 \end{bmatrix}$$

and

$$f(t) = \begin{bmatrix} 0 \\ 0 \\ \sin 2t \end{bmatrix}$$

as a single differential equation with initial conditions.

Name:________________________________

Table of Integrals

$$\int \sin^2 x\,dx = -\frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$$

$$\int \cos^2 x\,dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C$$

$$\int \sin^3 x\,dx = \frac{1}{12}\cos 3x - \frac{3}{4}\cos x + C$$

$$\int \cos^3 x\,dx = \frac{3}{4}\sin x + \frac{1}{12}\sin 3x + C$$

$$\int \left(\cos^2 x - \sin^2 x\right) dx = \frac{1}{2}\sin 2x + C$$