Ma 529

Homework #1

1. In each of the following find w_x and w_y

a) $w = e^x siny$

b) $w = e^x cosy$

c) $w = ln\sqrt{x^2 + y^2}$

d) $w = cosh(y/x^2)$

2. If $w = \sqrt{x^2 + y^2 + z^2}$, $x = e^r cos(s)$ $y = e^r sin(s)$, $z = e^s$, find w_r and w_s using the chain rule.

3. If a and b are constants and $w = (ax + by)^3 + tanh(ax + by) + cos(ax + by)$, show that

$$aw_y = bw_x.$$

4. If a and b are constants and $w = f(ax + by)$ is a differentiable function of $ax + by$, show that

$$aw_y = bw_x.$$

(Hint: Let $u = ax + by$ and apply the chain rule).

5. In each of the following verify that V satisfies Laplace's equation $V_{xx} + V_{yy} + V_{zz} = 0$.

a) $V = x^2 + y^2 - 2z^2$

b) $V = 2z^3 - 3(x^2 + y^2)z$

c) $V = ln\sqrt{x^2 + y^2}$

d) $V = e^{3x+4y} cos5z$

e) $V = (x^2 + y^2 + z^2)^{-1/2}$

6. If $I(a) = \int_0^1 \frac{x^a - x^b}{lnx}\, dx$ where $a > b > -1$ show that

$$I'(a) = \int_0^1 x^a dx = \frac{1}{a+1}$$

and hence, noticing that $I(b) = 0$, obtain the result

$$\int_0^1 \frac{x^a - x^b}{lnx}\, dx = \int_b^a \frac{du}{u+1} = ln\, \frac{a+1}{b+1} \qquad (a > b > -1).$$

(Here ln is the natural log).

7. Obtain a differential equation, together with appropriate initial conditions, satisfied by the function

$$y(x) = \frac{1}{2!} \int_a^x (x-t)^2 \; h(t)dt.$$