

# Homework 3

Due: Wednesday, February 18

## Problem 1

Three balls are to be randomly selected with replacement from an urn containing 20 balls numbered 1 through 20. If we bet that at least one of the drawn balls has a number as large as or larger than 17, what is the probability that we win the bet?

## Problem 2

Two balls are chosen randomly from an urn containing 8 white, 4 black, and 2 orange balls. Suppose that we win \$2 for each black ball selected and we lose \$1 for each white ball selected. Let  $X$  denote our winnings. What are the possible values of  $X$ , and what are the probabilities associated with each value?

### Problem 3

Suppose that a die is rolled twice. What are the possible values that the following random variables can take on? If the die is assumed fair, calculate the probabilities associated with the random variables.

- (a) The maximum value to appear in the two rolls.
- (b) The value of the first roll minus the value of the second roll.

### Problem 4

A filling station is supplied with gasoline once a week. If its weekly volume of sales in thousands of gallons is a random variable with probability density function

$$f(x) = \begin{cases} 5(1-x)^4, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

what need the capacity of the tank be so that the probability of the supply's being exhausted in a given week is .01?

## Problem 5

Consider the function

$$f(x) = \begin{cases} C(2x - x^3) & 0 < x < \frac{5}{2} \\ 0 & \text{otherwise} \end{cases}$$

Could  $f$  be a probability density function? If so, determine  $C$ . Repeat if  $f(x)$  were given by

$$f(x) = \begin{cases} C(2x - x^2) & 0 < x < \frac{5}{2} \\ 0 & \text{otherwise} \end{cases}$$

## Problem 6

You arrive at a bus stop at 10 o'clock, knowing that the bus will arrive at some time  $T$  between 10 and 10:30. The time  $T$ , measured in minutes after 10 o'clock, has the probability density

$$f(x) = \begin{cases} \frac{1}{30} & 0 \leq x \leq 30 \\ 0 & \text{elsewhere} \end{cases}$$

- (a) What is the probability that you will have to wait longer than 10 minutes?
- (b) If at 10:15 the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?