Homework 4

Due: Wednesday, February 25

Problem 1

Compute $E(X)$ if X has a density function given by

(a) $f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$

Hint: you will need integration by parts.

(b) $f(x) = \begin{cases} c(1 - x^2), & -1 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$

where c is a constant whose value you should determine from the properties of a probability density function.

Problem 2

The density function of X is given by

$$f(x) = \begin{cases} a + bx^2, & 0 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

If $E(X) = \frac{3}{5}$, find a and b.

Problem 3

A man aiming at a target receives 10 points if his shot is within 1 inch of the target, 5 points if it is between 1 and 3 inches of the target, and 3 points if it is between 3 and 5 inches of the target. Find the expected number of points scored if the distance from the shot to the target is a continuous random variable with probability density

$$f(x) = \begin{cases} \frac{1}{10}, & 0 \leq x \leq 10 \\ 0, & \text{elsewhere} \end{cases}$$

(A random variable with such probability density function is said to be *uniformly distributed between 0 and 10.*)

Problem 4

Let X be such that

$$P(X=1)=p \qquad P(X=-1)=1-p$$

Find $c \neq 1$ such that $E(c^X)=1$.

Problem 5

Two fair dice are rolled. Find the joint distribution of X and Y where X is the smallest and Y is the largest value obtained on the dice.

Problem 6

The joint probability density function of X and Y is given by

$$f(x,y) = \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) \qquad 0 < x < 1,\ 0 < y < 2$$

(If $x \leq 0$, $x \geq 1$, $y \leq 0$, or $y \geq 2$ then $f(x,y) = 0$)

(a) Compute the density function of X.

(b) Find $P(X > Y)$.