MA361 Partial Differential Equations

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Lectures: Mondays and Wednesdays, 10:00-11:15 am

Course webpage: http://personal.stevens.edu/~nstrigul/MA361/index_PDE.html

General comments:

MA361 is an undergraduate course in partial differential equations (PDEs). The major goal of this course is to study three types of PDEs—elliptic, hyperbolic, and parabolic—in a single spatial dimension, using both analytical and numerical methods. Specifically, we will carefully study how to deal with boundary value problems (BVPs) for the Laplace, wave, and heat equations. The course will also include necessary chapters of linear algebra and ordinary differential equations, and include an extensive computational part, employing the Mathematica software package. We will emphasize as much as possible the physical and geometrical ideas behind the theory, to keep results intuitively clear. The mandatory material will be the first seven chapters of Mark Gockenbach's book; this is the textbook which all students should buy. However, some lectures will be based on other texts, such as the first volume of R. Courant and D. Hilbert's classic "Methods of Mathematical Physics" and G. Strang's "Introduction to Applied Mathematics". Providing that the mandatory material is covered, we will proceed to consider modeling methods, such as scaling and nondimensionalisation, using the Navier-Stokes equations as an example.

Textbooks:

1) Partial Differential Equations: Analytical and Numerical Methods by M. S. Gockenbach, 2002.
2) Methods of Mathematical Physics by R. Courant, D. Hilbert, Volume 1.
3) Introduction to Applied Mathematics by G. Strang.
4) Equations of Mathematical Physics by V. S. Vladimirov.
5) Equations of Mathematical Physics by S. G. Mihlin.

Course program:

Jan 17. Lecture 1. **- Introduction. Classification of PDEs.**

Jan 22. Lecture 2. **- Derivation of the classical models 1.**

Jan 24. Lecture 3. **- Derivation of the classical models 2.**

Jan 29. Lecture 4. **- Essential linear algebra 1.**

Jan 31. Lecture 5. **- Essential linear algebra 2.**

Feb 5. Lecture 6. **- Essential ordinary differential equations 1.**

Feb 7. Lecture 7. **- Essential ordinary differential equations 2.**

Feb 12. Lecture 8. **- BVPs in statics 1. The analogy between BVPs and linear algebraic systems.**

Feb 14. Lecture 9. **- BVPs in statics 2. Introduction to the spectral method; eigenfunctions.**

Feb 20. Lecture 10. **- BVPs in statics 3. Solving the BVP using the Fourier series.**

Feb 21. Lecture 11. **- BVPs in statics 4. The Galerkin method.**

Feb 26. Lecture 12. **- Heat equation. Fourier series methods for the heat equation.**

Feb 28. Lecture 13. **- Pure Neumann conditions and the Fourier cosine series.**

Mar 5. Lecture 14. **- Periodic boundary conditions and the full Fourier series.**

Mar 7. Lecture 15. **- Finite element methods for the heat equation.**

Mar 19. Lecture 16. **- Finite elements and Neumann conditions.**

Mar 21. Lecture 17. **- Green's functions for the heat equation.**

Mar 26. Lecture 18. **- Waves. The homogeneous wave equation without boundaries.**

Mar 28. Lecture 19. **- Fourier series methods for the wave equation.**

Apr 2. Lecture 20. **- Finite element methods for the wave equation.**

Apr 4. Lecture 21. **- Point sources and resonance.**

Apr 9. Lecture 22. **- Physical models in two or three spatial dimensions 1. The heat equation.**

Apr 11. Lecture 23. **- Physical models in two or three spatial dimensions 2. The wave equation.**

Apr 16. Lecture 24. **- Physical models in two or three spatial dimensions 3. The Laplace's equation.**

Apr 18. Lecture 25. **- Advanced modeling 1. Derivation of the Navier-Stokes equations.**

Apr 23. Lecture 26. **- Advanced modeling 2. Nondimensionalisation of the Navier-Stokes equations.**

Apr 25. Lecture 27. **- Advanced modeling 3. Scaling of the Navier-Stokes equations.**