MA530. Homework 5

This part of the homework due October 4, 2006

Solve the following problems 1) analytically 2) using Mathematica

1. (#1 p.126) Use the Laplace transform to solve the initial value problem:

$$y' + 4y = 1;\ y(0) = -3$$

2. (#31 p.140) Use the Laplace transform to solve the initial value problem:

$$y'' + 4y = f(t);\ y(0) = 1;\ y'(0) = 0$$

with

$$f(t) = \begin{cases} 0, & \text{for } 0 \leqslant t < 4; \\ 3, & \text{for } t \geqslant 4. \end{cases}$$

3. (#7 p. 146) Use the convolution theorem to compute the inverse Laplace transform of the function (even if another method would work):

$$\frac{1}{s(s+2)} \exp^{-4s}$$

4. (#9 p. 146) Use the convolution theorem to write a formula for the solution of the initial value problem in terms of $f(t)$:

$$y'' - 5y' + 6y = f(t);\ y(0) = y'(0) = 0$$

5. (#1 p.151) Solve the initial value problem:

$$y'' + 5y' + 6y = 3\delta(t-2) - 4\delta(t-5);\ y(0) = y'(0) = 0$$

6. (#3 p. 156) Use the Laplace transform to solve the initial value problem for the system:

$$x' + 2y' - y = 1,\ 2x' + y = 0;\ x(0) = y(0) = 0$$

7. (#3 p. 161) Use the Laplace transform to solve the initial value problem:

$$y'' - 16ty' + 32y = 14; \; y(0) = y'(0) = 0$$

This part of the homework due October 25, 2006

1. (#1 p. 97) Find the general solution using the method of variation of parameters:

$$y'' + y = \tan(x)$$

2. (#7 p. 97) Find the general solution using the method of undetermined coefficients:

$$y'' - y' - 2y = 2x^2 + 5$$

3. (#13 p. 110) How many times can the bob pass through the equilibrium point in the case of overdamped motion? What condition can be placed on the initial displacement $y(0)$ to guarantee that the bob never passes through equilibrium?

4. (# 33 p. 111) Consider the damped forced motion governed by $my'' + cy' + ky = A\cos(\omega t)$. Show that the maximum amplitude of the steady-state solution is achieved if ω is chosen so that

$$\omega^2 = \frac{k}{m} - \frac{c^2}{2m^2}$$

In building a seismic detector, we would try to choose k, c, and m so that ω^2 is as near to this value as possible in order to maximize the response of the instrument.